(a) player 1 (or player 2) plays some given a with probability > 0.

**Extensive Games**

So far: only simultaneous, one-shot games

Q: How do we model sequential structures of many games (chain,...)?

A: Use extensive game (tree game)...

Then, players have several choice points when they can decide how to play. Strategies then map choice points to applicable actions.

**Definition:** An extensive game with perfect information (EGIPE) is a tuple $\Gamma = \langle N, A, H, P, (w_i)_{i\in N} \rangle$

where:

- $N$ is a finite, nonempty set of players.
- $A$ is a finite, nonempty set of actions.
- $H$ is a set of (finite or infinite) sequences over $A$ (called histories) and $H^*$:
  - the empty sequence $\langle \rangle \in H$,
  - if $\langle a_k \rangle_{k=1}^L \in H$ for some $K \in N \cup \{0\}$ and $L < K$ then $\langle a_k \rangle_{k=1}^L \in H$ (prefix closeness)
  - if $\langle a_k \rangle_{k=1}^\infty$ is an action sequence such that $\langle a_k \rangle_{k=1}^L \in H$ for all $L \in N$, then $\langle a_k \rangle_{k=1}^\infty \in H$.

A history is called terminal if it is infinite or if it is not a prefix of any longer history. The set of terminal histories is denoted by $T$. $P : H \setminus T \rightarrow N$ is the play function assigning to each nonterminal history $h \in H \setminus T$ a player $P(h)$ whose turn it is to move in $h$. For each player $i \in N$, $w_i : Z \rightarrow R$ is his utility function.

**Terminology:**
- $\Gamma$ is finite if $H$ is finite.
- $\Gamma$ has a finite horizon if $H$ contains no infinite histories.
Example (Shapley game):

Task: Two players have to share two identical objects.
- Player 1 proposes an allocation.
- Player 2 accepts or declines.
  If objects are divided, move goes as proposed.

We have:

\[ A = P(\{S\}) \]

\[ Z = P(\{(1,2)\}) \]

\[ \mu_{1}(\{(2,0), y\}) = 2 \]

Finally:

\[ \Gamma = \langle N, \Omega, H, P, \{u_{i}\}_{i=1}^{n} \rangle \]

\[ N = \{1, 2\}, \quad \Omega = \{(2,0), (1,1), (0,2), y, n\} \]

\[ H = \{(\gamma), (2,0), (1,1), (0,2), (2,0), y, \ldots\} \]

\[ P(\gamma) = 1, \quad P(\{\} = 2 \quad \text{for all } \{\} \in H \setminus \{\gamma\} \]

\[ \mu_{1}(\{(2,0), y\}) = 2, \quad \text{etc.} \]