Prisoner's dilemma

Setting: Two prisoners held in separate cells;
- if both confess, they will be sentenced to 3 years of prison;
- if neither confesses, they will be sentenced to 1 year;
- if only one confesses (denies guilt), then
  the other one gets a sentence of four years.

Battle of Sexes (Dax or Struvinsky)

A coordination game

Setting: Two people want to go together to a concert. They made an appointment to meet, but forget to choose the concert. Player A likes Bach and player B Struvinsky. Both want to be together.

<table>
<thead>
<tr>
<th></th>
<th>Bach</th>
<th>Struvinsky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bach</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>Struvinsky</td>
<td>0,0</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Dove-Hawk or Chicken-game

Setting: Two agents are competing for a resource. They can decide to act like a hawk (attacking) or like a dove (yielding). If both attack, the outcome will be zero. If both yield, they get a moderate payoff, otherwise the hawk gets a high payoff and the dove a little bit.

<table>
<thead>
<tr>
<th></th>
<th>Hawk</th>
<th>Dove</th>
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</thead>
<tbody>
<tr>
<td>Hawk</td>
<td>0,0</td>
<td>4,1</td>
</tr>
<tr>
<td>Dove</td>
<td>1,4</td>
<td>2,2</td>
</tr>
</tbody>
</table>

Anti-coordination game
Matching Pennies

Setting: 2 people have to make a choice between "head" and "tail."
If choices differ, then person 1 gives 1 to 2.
If choices are the same, then person 2 gives 1 to 1.

<table>
<thead>
<tr>
<th></th>
<th>H</th>
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<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Example of a zero sum game, i.e., the sum of the utilities for all strategy profiles is zero.
Zero sum games are a special case of strictly competitive games: \( U_A(a) \leq U_A(b) \implies U_2(a) \geq U_2(b) \).

Given strategy profile \( \alpha = (a_1, \ldots, a_M) \), let \( \bar{a} \in A_i \) then \( (a_{-i}, \bar{a}_i) = (a_1, \ldots, a_{i-1}, \bar{a}_i, a_{i+1}, \ldots, a_M) \).

Example:

\[
A_1 = \{T, B\}, \quad A_2 = \{L, R\}, \quad A_3 = \{X, Y, Z\}
\]

\(a = (T, R, Z), \quad a_2 = (T, Z), \quad a_3 = (T, R)\)

\((a_2, L) = (T, L, Z)\)

2.4 Solution Concepts & Notation

- What is a "solution" of a game? → strategy profile, where all players play strategies that are rational.

When talking/writing strategy profiles, we often want to talk about a profile, where one player is removed or whose original strategy has been replaced.

**Notation:** Let \( \alpha = (a_1, \ldots, a_M) \in \prod A_i = A \)

\( A_{-i} = \prod A_{i}, \bar{a} \in \prod A_{-i} = \{(), (), ()\} \)

\( q_i = (a_{i1}, a_{i2}, \ldots, a_{iM}) \)

2.5 Elimination of Dominated Strategies

- What strategy should the agent play? → Eliminate all irrational strategies.

**Def:** A strategy \( a_i \in A_i \) is strictly dominated by strategy \( a^* \in A_i \) if for all profiles of the other agents

\( a_{-i} \in A_{-i} : U_i(a_{-i}, a^*_i) < U_i(a_{-i}, a_i)\).

\[
\begin{array}{ccc}
\text{Conf.} & \text{Don't} \\
\hline
\text{Conf.} & 1 & 4 & 0 \\
\text{Don't} & 0 & 3 & 2 \\
\end{array}
\]

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
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<td>0, 0</td>
</tr>
<tr>
<td>S</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>
**Definition**

A strategy \( a^*_i \in A_i \) is weakly dominated by \( a^*_i \in A_i \) if for each \( a_{-i} \in A_{-i} \):

\[
U_i(a_{-i}, a^*_i) \leq U_i(a_{-i}, a^*_i)
\]

and if for our profile \( a_{-i} \in A_{-i} \):

\[
U_i(a_{-i}, a^*_i) < U_i(a_{-i}, a^*_i).
\]

---

2.6 Nash Equilibrium

**Alternative Def**

\( B_i(a_{-i}) = \{ a \in A_i \mid U_i(a_{-i}, a) \geq U_i(a_{-i}, a') \quad \forall a' \in A_i \} \)

\( B_i : A_i \rightarrow A_i \) is called best response function

A NE \( a^* \) is a profile

\( a^*_i \in B_i(a_{-i}) \)

i.e. \( a^* \in \cap_i B_i(a_{-i}) = B(a^*) \)

\( a^* \in B(a^*) \)

\( f(x) = x \text{ for all } x \)

---

2.6 Nash Equilibrium

**Alternative Solution concept:** Focus on "stable" profiles - a profile where for each agent it does not make sense to deviate.

**Def (Nash Equilibrium)**

A Nash equilibrium (NE) of a strategic game \( G \) is a strategy profile \( a^* \in A \) such that for each \( i \in H \):

\[
U_i(a^*) \geq U_i(a^*_i, a^*_i) \quad \text{for all } a_i \in A_i.
\]