Exercise 6.1 (Planning in the wumpus world)

Consider the following initial state in the wumpus world:

The agent in square [1, 1] did not attend the “Action Planning” lecture, thus, he isn’t able to solve planning tasks with partial observability. Additionally, he is more excited about hunting the wumpus than about finding gold. Therefore, we define the planning problem as:

Initial state $I$:

\[
\text{connected}([1, 1], [2, 1]), \text{connected}([2, 1], [3, 1]), \ldots, \\
\text{connected}([4, 3], [4, 4]), \text{at}(\text{agent}, [1, 1]), \text{at}(\text{wumpus}, [1, 3]), \\
\text{at}(\text{pit}, [3, 1]), \text{at}(\text{pit}, [4, 4]), \text{arrowleft}, \text{agent\_alive}
\]

Operators $O$:

Move($x, y$)

PRE : $\text{at}(\text{agent}, x) \land \text{connected}(x, y) \land \text{agent\_alive}$

EFF : $\text{at}(\text{wumpus}, y) \triangleright \neg \text{agent\_alive},$

\[
\text{at}(\text{pit}, y) \triangleright \neg \text{agent\_alive}, \\
\text{at}(\text{agent}, y), \\
\neg \text{at}(\text{agent}, x)
\]

Shoot($x, y$)

PRE : $\text{at}(\text{agent}, x) \land \text{connected}(x, y) \land \text{arrowleft} \land \text{agent\_alive}$

EFF : $\text{at}(\text{wumpus}, y) \triangleright \text{scream},$

\[
\neg \text{arrowleft}
\]

$^{1}$ *stench*, *breeze* and *gold* will not be formalized here and serve only for the purpose of illustration (or confusion?).
Goal $G$: 

$scream \land agent\_alive$

(a) Suppose, you want to solve a simplified, monotonic planning problem by ignoring negative effects (aka. the “delete relaxation”) in order to calculate a heuristic. Specify the operators of the relaxed planning task.

(b) Sketch the first two levels of the relaxed planning graph. Facts that do not change in the relaxed problem, e.g. $agent\_alive$, $at(pit, x)$ and $connected(x, y)$ can be omitted (In the initial state in layer $F0$ you only have to sketch the fact $at(agent, [1, 1])$). To further simplify the problem, you may compile away the conditional effect $at(wumpus, y) \triangleright scream$ of $Shoot(x, y)$ by moving the effect precondition to the operator precondition$^2$.

(c) Contrary to the PlanGraph method presented in the lecture, actions cannot be conflicting in a relaxed planning problem since they neither contain negative preconditions nor negative effects. Therefore, relaxed plans can be found more easily and thus be used to derive heuristic estimates. Specify the relaxed plan. Is this plan also applicable in the original problem?

Exercise 6.2 (Bayesian Networks)

Consider the following Bayesian network:

![Bayesian Network Diagram]

(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network ($Ind(U, V \mid W)$ denotes that $U$ is conditionally independent of $V$ given $W$, and $Ind(U, V)$ denotes unconditional independence of $U$ and $V$):

(i) $Ind(Cold, Winter)$

(ii) $Ind(Winter, NegligentDriver)$

(iii) $Ind(Winter, RadioSilent \mid BatteryProblem)$

(iv) $Ind(Winter, EngineNotStarting \mid BatteryProblem)$

(v) $Ind(Cold, NegligentDriver \mid RadioSilent)$

$^2$When compiling away conditional effects, usually two operators (one with the effect condition and one with the negated effect condition) are created. However, $Shoot'(x, y) = (PRE: at(agent, x), \neg at(wumpus, y), \ldots \ EFF: \emptyset)$ does not have any effect and might be excluded here as a result.
(b) Compute $P(\text{EngineNotStarting}|\text{NegligentDriver}, \neg\text{Cold})$. The relevant entries in the conditional probability tables are given below:

\[
\begin{align*}
P(\text{LightsOnOverNight}|\text{NegligentDriver}) &= 0.3 \\
P(\text{LightsOnOverNight}|\neg\text{NegligentDriver}) &= 0.02 \\
P(\text{TankEmpty}|\text{NegligentDriver}) &= 0.1 \\
P(\text{TankEmpty}|\neg\text{NegligentDriver}) &= 0.01 \\
P(\text{BatteryProblem}|\text{Cold, LightsOnOverNight}) &= 0.9 \\
P(\text{BatteryProblem}|\text{Cold, } \neg\text{LightsOnOverNight}) &= 0.2 \\
P(\text{BatteryProblem}|\neg\text{Cold, LightsOnOverNight}) &= 0.8 \\
P(\text{BatteryProblem}|\neg\text{Cold, } \neg\text{LightsOnOverNight}) &= 0.01 \\
P(\text{EngineNotStarting}|\text{BatteryProblem, TankEmpty}) &= 0.9 \\
P(\text{EngineNotStarting}|\neg\text{BatteryProblem, TanksEmpty}) &= 0.7 \\
P(\text{EngineNotStarting}|\neg\text{BatteryProblem, TankEmpty}) &= 0.8 \\
P(\text{EngineNotStarting}|\neg\text{BatteryProblem, TankEmpty}) &= 0.05
\end{align*}
\]

Exercise 6.3 (Bayesian Rule)
Assume, you are at night in Athens and witness a car accident in which a taxi is involved. 90% of the taxis in Athens are green, all others are blue. You are absolutely sure that the taxi involved in the accident was blue. But tests show that distinguishing between blue and green at darkness is only 75% reliable. If you take this into consideration, what is the probability that the taxi was really blue? (Hint: Distinguish exactly between the statement that a taxi is red and the statement that a taxi appears to be red.)

Exercise 6.4 (Conditional independence)
This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $P(X|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

(i) $P(E_1, E_2), P(X), P(E_1|X), P(E_2|X)$
(ii) $P(E_1, E_2), P(X), P(E_1, E_2|X)$
(iii) $P(X), P(E_1|X), P(E_2|X)$

(b) Suppose we know that $P(E_1|X, E_2) = P(E_1|X)$ for all values of $X$, $E_1$, and $E_2$. Now which of the three sets are sufficient?