Exercise 4.1 (Satisfiability and Resolution)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.
   
   (i) $Smoke \Rightarrow Smoke$
   
   (ii) $Smoke \Rightarrow \neg Smoke$
   
   (iii) $Smoke \Rightarrow Fire$
   
   (iv) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Fire \Rightarrow \neg Smoke)$
   
   (v) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)$
   
   (vi) $Spring \Leftrightarrow SunnyWeather$

(b) Let $K = \{\{A, B, \neg C\}, \{\neg A, C\}, \{\neg A, \neg B\}, \{A, C\}\}$. Use the resolution method to show that $K \models (\neg B \Rightarrow (A \land C))$.

Exercise 4.2 (Davis-Putnam Procedure)

Use the Davis-Putnam procedure to compute models for the following clause sets or to prove that no model exists. Whenever possible, apply unit propagation. At each step, indicate which rule you have applied.

(a) $\{\{P, \neg Q\}, \{-P, Q\}, \{Q, \neg R\}, \{S\}, \{-S, \neg Q, \neg R\}, \{S, R\}\}$

(b) $\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\}$
Exercise 4.3 (Wumpus world and resolution)

Consider the following situation in the wumpus world:

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1,1 2,1 3,1 4,1
1,2 2,2 3,2 4,2
1,3 2,3 3,3 4,3
1,4 2,4 3,4 4,4
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The gray squares have already been visited, the others not. The percepts in the corresponding squares are indicated by \textit{breeze} and \textit{stench}.

(a) Formalize the general connections between breezes and pits using propositional formulae. Use 16 variables \(P_{i,j}\) (meaning there is a pit in square \([i, j]\)) and 16 variables \(B_{i,j}\) (breeze in square \([i, j]\)).

(b) Show, using resolution, that square \([3, 1]\) contains a pit in the given situation, i.e., show that \(\text{KB} \models P_{3,1}\). The knowledge base \(\text{KB}\) consists of the propositions from part (a) as well as the percepts of the agent. Note: squares that already have been visited do not contain pits. If necessary, convert the knowledge base into CNF (conjunctive normal form).

Exercise 4.4 (Predicate Logic)

Consider following colloquial sentences:

(a) Not all students attend AI and ST.

(b) One student failed both AI and ST.

(c) Exactly two students failed ST.

(d) There is a barber who shaves all men in town who do not shave themselves.

(e) No one likes a professor who is not smart.

Represent these sentences in first-order logic using the predicates \(\text{student}(x)\), \(\text{attends}(x,y)\), \(\text{fails}(x,y)\), \(\text{barber}(x)\), \(\text{shaves}(x,y)\), \(\text{professor}(x)\), \(\text{likes}(x,y)\) und \(\text{smart}(x)\).

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.