Submission: hand in by 18th May 2011 before 16:00

- The solutions should be submitted in English.
- You must work on your own and write down your own solution. This does not exclude occasional discussions with your fellow students, but solutions copied from other students will not be accepted.

Exercise 1.2 - Internal Path Length

The internal path length $l(t)$ of a search tree $t$ is defined as follows:

$$
l(t) = \begin{cases} 
0 & \text{if } t \text{ is empty} \\
l(t_l) + l(t_r) + \text{size}(t) & \text{otherwise}
\end{cases}
$$

where $t_l, t_r$ are respectively the left and right subtrees of $t$ and $\text{size}(t)$ denotes the number of internal nodes of $t$. Now, let $N(t)$ denote the internal nodes of $t$. Using induction show that:

$$l(t) = \sum_{p \in N(t)} (\text{depth}(p) + 1)$$

where $\text{depth}(p)$ is the distance of node $p$ from the root of $t$.

Hint: For the base case consider the empty tree. Then, for the inductive step, assume that for a given tree $t$ the desired property holds for subtrees $t_l$ and $t_r$. Notice that $\text{depth}(p)$ has different values depending on the given tree. Assigning different names might be helpful (e.g. $\text{depth}_1, \text{depth}_{t_l}, \text{depth}_{t_r}$).

Exercise 1.3 - Trees

Consider the following Binary Search Tree:

1. Which sequences (permutations) of the keys 1,2,3,4 will result in this shape, if keys are inserted sequentially in an empty tree?
2. Draw all structurally different binary trees with four internal nodes. (Do not draw the leaf nodes).
3. Derive the formula for the number $B_N$ of structurally different binary trees with $N$ internal nodes. Explain your solution.