Principles of Knowledge Representation and Reasoning
Semantic Networks and Description Logics II:
Description Logics – Terminology and Notation

Bernhard Nebel, Stefan Wölfl, and Marco Ragni

Albert-Ludwigs-Universität Freiburg

July 5, 2010
Description Logics – Terminology and Notation

Introduction

Concept and Roles

TBox and ABox

Reasoning Services

Outlook
Motivation

- Main problem with semantic networks and frames
- The lack of formal semantics!
- Disadvantage of simple inheritance networks
- Concepts are atomic and do not have any structure

Brachman’s structural inheritance networks (1977)
Structural Inheritance Networks

- Concepts are defined/described using a small set of well-defined operators
- Distinction between conceptual and object-related knowledge
- Computation of subconcept relation and of instance relation
- Strict inheritance (of the entire structure of a concept)
Systems and Applications

- **Systems:**
  - KL-ONE: First implementation of the ideas (1978)
  - ... then NIKL, KL-TWO, KRYPTON, KANDOR, CLASSIC, BACK, KRIS, YAK, CRACK ...
  - ... currently FaCT, DLP, RACER 1998

- **Applications:**
  - First, natural language understanding systems
  - ... then configuration systems,
  - ... information systems,
  - ... currently, it is one tool for the *semantic web*
  - DAML+OIL, now OWL
Description Logics

- Previously also *KL-ONE-alike languages, frame-based languages, terminological logics, concept languages*

- Description Logics (DL) allow us
  - to describe concepts using *complex descriptions*,
  - to introduce the terminology of an application and to structure it (TBox),
  - to introduce objects (ABox) and relate them to the introduced terminology,
  - and to *reason* about the terminology and the objects.
Informal Example

**Male** is: the opposite of female

A **human** is a kind of: living entity

A **woman** is: a human and a female

A **man** is: a human and a male

A **mother** is: a woman with at least one child that is a human

A **father** is: a man with at least one child that is a human

A **parent** is: a mother or a father

A **grandmother** is: a woman, with at least one child that is a parent

A **mother-wod** is: a mother with only male children

Possible Questions:

- Is a grandmother a parent?
- Is Diana a parent?
- Is William a man?
- Is Elizabeth a mother-wod?
Atomic Concepts and Roles

- **Concept names:**
  - E.g., Grandmother, Male, ... (in the following usually *capitalized*)
  - We will use *symbols* such as $A, A_1, ...$
  - *Semantics:* Monadic predicates $A(\cdot)$ or set-theoretically a subset of the universe $A^I \subseteq D$.

- **Role names:**
  - In our example, e.g., child. Often we will use names such as *has-child* or something similar (in the following usually *lowercase*).
  - Role names are *disjoint* from concept names
  - *Symbolically:* $t, t_1, ...$
  - *Semantics:* Dyadic predicates $t(\cdot, \cdot)$ or set-theoretically $t^I \subseteq D \times D$. 
Concept and Role Description

- Out of *concept* and *role names*, complex *descriptions* can be created.
- In our example, e.g. “a Human and Female.”
- Symbolically: *C* for concept descriptions and *r* for role descriptions.

Which particular constructs are available depends on the chosen description logic!

- Predicate logic semantics: A concept descriptions *C* corresponds to a formula *C*(*)x* with the free variable *x*. Similarly with *r*: It corresponds to formula *r*(*)x, y* with free variables *x, y*.

- Set semantics:

\[
C^I = \{ d \mid C(d) \text{ “is true in” } I \} \\
r^I = \{ (d, e) \mid r(d, e) \text{ “is true in” } I \}
\]
Boolean Operators

- **Syntax**: let $C$ and $D$ be concept descriptions, then the following are also concept descriptions:
  - $C \cap D$ (concept conjunction)
  - $C \cup D$ (concept disjunction)
  - $\neg C$ (concept negation)

- **Examples**:
  - Human $\cap$ Female
  - Father $\sqcap$ Mother
  - $\neg$ Female

- **Predicate logic semantics**: $C(x) \land D(x)$, $C(x) \lor D(x)$, $\neg C(x)$

- **Set semantics**: $C^I \cap D^I$, $C^I \cup D^I$, $D \setminus C^I$
Role Restrictions

- **Motivation:**
  - Often we want to describe something by *restricting* the possible “fillers” of a role, e.g. Mother-wod.
  - Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother

- **Idea:** Use quantifiers that range over the role-fillers
  - Mother $\sqcap \forall \text{has-child}. \text{Man}$
  - Woman $\sqcap \exists \text{has-child}. \text{Parent}$

- **Predicate logic semantics:**

  \[
  (\exists r. C)(x) = \exists y : (r(x, y) \land C(y)) \\
  (\forall r. C)(x) = \forall y : (r(x, y) \rightarrow C(y))
  \]

- **Set semantics:**

  \[
  (\exists r. C)^I = \{ d | \exists e : (d, e) \in r^I \land e \in C^I \} \\
  (\forall r. C)^I = \{ d | \forall e : (d, e) \in r^I \rightarrow e \in C^I \}
  \]
Cardinality Restriction

- **Motivation:**
  - Often we want to describe something by *restricting the number* of possible “fillers” of a role, e.g., a Mother with at least 3 children or at most 2 children.

- **Idea:** We restrict the cardinality of the role filler sets:
  - Mother ∩ (≥ 3 has-child)
  - Mother ∩ (≤ 2 has-child)

- **Predicate logic semantics:**
  \[
  (\geq n r)(x) = \exists y_1 \ldots y_n : (r(x, y_1) \land \ldots \land r(x, y_n) \land \\
  y_1 \neq y_2 \land \ldots \land y_{n-1} \neq y_n)
  \]
  \[
  (\leq n r)(x) = \neg (\geq n + 1 r)(x)
  \]

- **Set semantics:**
  \[
  (\geq n r)^I = \{ d \mid |\{ e \mid r^I(d, e)\}| \geq n \}
  \]
  \[
  (\leq n r)^I = D - (\geq n + 1 r)^I
  \]
Inverse Roles

- **Motivation:**
  - How can we describe the concept “children of rich parents”?

- **Idea:** Define the “inverse” role for a given role (the converse relation)
  - has-child$^{-1}$

- **Application:** $\exists$has-child$^{-1}$.Rich

- **Predicate logic semantics:**
  \[
  r^{-1}(x, y) = r(y, x)
  \]

- **Set semantics:**
  \[
  (r^{-1})^I = \{(d, e) \mid (e, d) \in r^I\}
  \]
Role Composition

- **Motivation:**
  - How can we define the role `has-grandchild` given the role `has-child`?

- **Idea:** Compose roles (as one can compose binary relations)
  - `has-child` ◦ `has-child`

- **Predicate logic semantics:**
  \[
  (r \circ s)(x, y) = \exists z : (r(x, z) \land s(z, y))
  \]

- **Set semantics:**
  \[
  (r \circ s)^T = \{(d, e) \mid \exists f : (d, f) \in r^T \land (f, e) \in s^T\}
  \]
Role Value Maps

- **Motivation:**
  - How do we express the concept “women who know all the friends of their children”

- **Idea:** Relate role filler sets to each other
  - Woman $\sqcap (\text{has-child} \circ \text{has-friend} \sqsubseteq \text{knows})$

- **Predicate logic semantics:**
  \[
  (r \sqsubseteq s)(x) = \forall y : (r(x, y) \rightarrow s(x, y))
  \]

- **Set semantics:** Let $r^\mathcal{I}(d) = \{ e | r^\mathcal{I}(d, e) \}$.
  \[
  (r \sqsubseteq s)^\mathcal{I} = \{ d | r^\mathcal{I}(d) \subseteq s^\mathcal{I}(d) \}
  \]

- **Note:** Role value maps lead to undecidability of satisfiability of concept descriptions!
In order to *introduce* new terms, we use two kinds of *terminological axioms*:

- $A \equiv C$
- $A \sqsubseteq C$

where $A$ is a *concept name* and $C$ is a *concept description*.

A *terminology* or *TBox* is a finite set of such axioms with the following additional restrictions:

- no multiple definitions of the same symbol such as $A \equiv C$, $A \sqsubseteq D$
- no cyclic definitions (even not indirectly), such as $A \equiv \forall r . B$, $B \equiv \exists s . A$
TBoxes: Semantics

- TBoxes restrict the set of possible interpretations.

- **Predicate logic semantics:**
  - $A \equiv C$ corresponds to $\forall x : (A(x) \leftrightarrow C(x))$
  - $A \sqsubseteq C$ corresponds to $\forall x : (A(x) \rightarrow C(x))$

- **Set semantics:**
  - $A \equiv C$ corresponds to $A^\mathcal{I} = C^\mathcal{I}$
  - $A \sqsubseteq C$ corresponds to $A^\mathcal{I} \subseteq C^\mathcal{I}$

- Non-empty interpretations which satisfy all terminological axioms are called models of the TBox.
Assertional Box

- In order to state something about objects in the world, we use two forms of assertions:
  - $a : C$
  - $(a, b) : r$

  where $a$ and $b$ are **individual names** (e.g., ELIZABETH, PHILIP), $C$ is a **concept description**, and $r$ is a **role description**.

- An **ABox** is a finite set of assertions.
ABoxes: Semantics

- **Individual names** are interpreted as elements of the universe under the unique-name-assumption, i.e., different names refer to different objects.
- **Assertions** express that an object is an instance of a concept or that two objects are related by a role.
- **Predicate logic semantics:**
  - $a : C$ corresponds to $C(a)$
  - $(a, b) : r$ corresponds to $r(a, b)$
- **Set semantics:**
  - $a^I \in D$
  - $a : C$ corresponds to $a^I \in C^I$
  - $(a, b) : r$ corresponds to $(a^I, b^I) \in r^I$
- **Models** of an ABox and of $\text{ABox} + \text{TBox}$ can be defined analogously to models of a TBox.
Example TBox

\[
\begin{align*}
\text{Male} & \equiv \neg \text{Female} \\
\text{Human} & \sqsubseteq \text{Living\_entity} \\
\text{Woman} & \equiv \text{Human} \sqcap \neg \text{Female} \\
\text{Man} & \equiv \text{Human} \sqcap \neg \text{Male} \\
\text{Mother} & \equiv \text{Woman} \sqcap \exists \text{has\_child}.\text{Human} \\
\text{Father} & \equiv \text{Man} \sqcap \exists \text{has\_child}.\text{Human} \\
\text{Parent} & \equiv \text{Father} \sqcup \text{Mother} \\
\text{Grandmother} & \equiv \text{Woman} \sqcap \exists \text{has\_child}.\text{Parent} \\
\text{Mother\_without\_daughter} & \equiv \text{Mother} \sqcap \forall \text{has\_child}.\text{Male} \\
\text{Mother\_with\_many\_children} & \equiv \text{Mother} \sqcap (\geq 3 \text{has\_child})
\end{align*}
\]
Example ABox

CHARLES: Man
EDWARD: Man
ANDREW: Man
DIANA: Woman
ELIZABETH: Woman

(DIANA, WILLIAM): has-child
(CHARLES, WILLIAM): has-child
(ELIZABETH, CHARLES): has-child
(ELIZABETH, EDWARD): has-child
(ELIZABETH, ANDREW): has-child
DIANA: Mother-without-daughter
Some Reasoning Services

- Does a description $C$ make sense at all, i.e., is it satisfiable?
- A concept description $C$ is satisfiable iff there exists an interpretation $I$ such that $C^I \neq \emptyset$.
- Is one concept a specialization of another one, is it subsumed?
  - $C$ is subsumed by $D$, in symbols $C \subseteq D$ iff we have for all interpretations $C^I \subseteq D^I$.
- Is $a$ an instance of a concept $C$?
  - $a$ is an instance of $C$ iff for all interpretations, we have $a^I \in C^I$.
- **Note**: These questions can be posed with or without a TBox that restricts the possible interpretations.
Can we reduce the reasoning services to perhaps just one problem?
What could be reasoning algorithms?
What about complexity and decidability?
What has all that to do with modal logics?
How can one build efficient systems?
Literature


## Summary: Concept Descriptions

<table>
<thead>
<tr>
<th>Abstract</th>
<th>Concrete</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>$A^I$</td>
</tr>
<tr>
<td>$C \cap D$</td>
<td>(and $C \cap D$)</td>
<td>$C^I \cap D^I$</td>
</tr>
<tr>
<td>$C \cup D$</td>
<td>(or $C \cup D$)</td>
<td>$C^I \cup D^I$</td>
</tr>
<tr>
<td>$\neg C$</td>
<td>(not $C$)</td>
<td>$D - C^I$</td>
</tr>
<tr>
<td>$\forall r. C$</td>
<td>(all $r$ $C$)</td>
<td>${d \in D : r^I(d) \subseteq C^I}$</td>
</tr>
<tr>
<td>$\exists r. C$</td>
<td>(some $r$ $C$)</td>
<td>${d \in D : r^I(d) \neq \emptyset}$</td>
</tr>
<tr>
<td>$\geq n r$</td>
<td>(atleast $n$ $r$)</td>
<td>${d \in D :</td>
</tr>
<tr>
<td>$\leq n r$</td>
<td>(atmost $n$ $r$)</td>
<td>${d \in D :</td>
</tr>
<tr>
<td>$\exists r. C$</td>
<td>(some $r$ $C$)</td>
<td>${d \in D : r^I(d) \cap C^I \neq \emptyset}$</td>
</tr>
<tr>
<td>$\geq n r. C$</td>
<td>(atleast $n$ $r$ $C$)</td>
<td>${d \in D :</td>
</tr>
<tr>
<td>$\leq n r. C$</td>
<td>(atmost $n$ $r$ $C$)</td>
<td>${d \in D :</td>
</tr>
<tr>
<td>$r = s$</td>
<td>(eq $r$ $s$)</td>
<td>${d \in D : r^I(d) = s^I(d)}$</td>
</tr>
<tr>
<td>$r \neq s$</td>
<td>(neq $r$ $s$)</td>
<td>${d \in D : r^I(d) \neq s^I(d)}$</td>
</tr>
<tr>
<td>$r \subseteq s$</td>
<td>(subset $r$ $s$)</td>
<td>${d \in D : r^I(d) \subseteq s^I(d)}$</td>
</tr>
<tr>
<td>$g = h$</td>
<td>(eq $g$ $h$)</td>
<td>${d \in D : g^I(d) = h^I(d) \neq \emptyset}$</td>
</tr>
<tr>
<td>$g \neq h$</td>
<td>(neq $g$ $h$)</td>
<td>${d \in D : \emptyset \neq g^I(d) \neq h^I(d) \neq \emptyset}$</td>
</tr>
<tr>
<td>${i_1, i_2, \ldots, i_n}$</td>
<td>(oneof $i_1 \ldots i_n$)</td>
<td>${i_1^I, i_2^I, \ldots, i_n^I}$</td>
</tr>
</tbody>
</table>
### Summary: Role Descriptions

<table>
<thead>
<tr>
<th>Abstract</th>
<th>Concrete</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>$t^I$</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>$f^I$, (functional role)</td>
</tr>
<tr>
<td>$r \cap s$</td>
<td>(and $r , s$)</td>
<td>$r^I \cap s^I$</td>
</tr>
<tr>
<td>$r \cup s$</td>
<td>(or $r , s$)</td>
<td>$r^I \cup s^I$</td>
</tr>
<tr>
<td>$\neg r$</td>
<td>(not $r$)</td>
<td>$D \times D - r^I$</td>
</tr>
<tr>
<td>$r^{-1}$</td>
<td>(inverse $r$)</td>
<td>${(d, d') : (d', d) \in r^I}$</td>
</tr>
<tr>
<td>$r</td>
<td>_C$</td>
<td>(restr $r , C$)</td>
</tr>
<tr>
<td>$r^+$</td>
<td>(trans $r$)</td>
<td>$(r^I)^+$</td>
</tr>
<tr>
<td>$r \circ s$</td>
<td>(compose $r , s$)</td>
<td>$r^I \circ s^I$</td>
</tr>
<tr>
<td>$1$</td>
<td>self</td>
<td>${(d, d) : d \in D}$</td>
</tr>
</tbody>
</table>