Principles of Knowledge Representation and Reasoning

Nonmonotonic Reasoning II: Minimal Models and Nonmonotonic Logic Programs

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Conflicts between defaults in default logic lead to multiple extensions.

Each extension corresponds to a maximal set of non-violated defaults.

Reasoning with defaults can also be achieved by a simpler mechanism: predicate or propositional logic + minimize the number of cases where a default (expressed as a conventional formula) is violated.

\[ \Rightarrow \text{minimal models} \]

Notion of \textit{minimality}: cardinality vs. set-inclusion.
Entailment with respect to Minimal Models

Definition

Let $A$ be a set of atomic propositions. Let $\Phi$ be a set of propositional formulae on $A$, and $B \subseteq A$ a set (called abnormalities).

Then $\psi$ $B$-minimally follows from $\Phi$ ($\Phi \models_B \psi$) if $\mathcal{I} \models \psi$ for all interpretations $\mathcal{I}$ such that

- $\mathcal{I} \models \Phi$ and
- there is no $\mathcal{I}'$ such that $\mathcal{I}' \models \Phi$ and $\{b \in B \mid \mathcal{I}' \models b\} \subsetneq \{b \in B \mid \mathcal{I} \models b\}$. 
Minimal models: example

\[ \Phi = \left\{ \begin{array}{l}
\text{student} \land \neg \text{ABstudent} \rightarrow \neg \text{earnsmoney}, \\
\text{student}, \\
\text{adult} \land \neg \text{ABadult} \rightarrow \text{earnsmoney}, \\
\text{student} \rightarrow \text{adult}
\end{array} \right\} \]

\( \Phi \) has the following models:

\[ \mathcal{I}_1 \models \text{student} \land \text{adult} \land \text{earnsmoney} \land \text{ABstudent} \land \text{ABadult} \]
\[ \mathcal{I}_2 \models \text{student} \land \text{adult} \land \neg \text{earnsmoney} \land \text{ABstudent} \land \text{ABadult} \]
\[ \mathcal{I}_3 \models \text{student} \land \text{adult} \land \text{earnsmoney} \land \text{ABstudent} \land \neg \text{ABadult} \]
\[ \mathcal{I}_4 \models \text{student} \land \text{adult} \land \neg \text{earnsmoney} \land \neg \text{ABstudent} \land \text{ABadult} \]
Relation to Default Logic

We can embed propositional minimal model reasoning in the propositional default logic.

**Theorem**

Let $A$ be a set of atomic propositions. Let $\Phi$ be a set of propositional formulae on $A$, and $B \subseteq A$. Then $\Phi \models_B \psi$ if and only if $\psi$ follows from $\langle D, W \rangle$ skeptically, where

$$D = \left\{ \frac{\neg b}{\neg b} \mid b \in B \right\} \text{ and } W = \Phi.$$
Proof sketch.

“⇒”: Assume there is an extension $E$ of $\langle D, W \rangle$ such that $\psi \not\in E$. Hence there is an interpretation $\mathcal{I}$ such that $\mathcal{I} \models E$ and $\mathcal{I} \models \neg \psi$. By the fact that there is no extension $F$ such that $E \subset F$, $\mathcal{I}$ is a $B$-minimal model of $\Phi$. Hence $\psi$ does not $B$-minimally follow from $\Phi$.

“⇐”: Assume $\psi$ does not $B$-minimally follow from $\Phi$. Hence there is a $B$-minimal model $\mathcal{I}$ of $\Phi$ such that $\mathcal{I} \not\models \psi$. Define

$$E = \text{Th}(\Phi \cup \{\neg b \models b \in B, \mathcal{I} \models \neg b\}).$$

Now $\mathcal{I} \models E$ and because $\mathcal{I} \not\models \psi$, $\psi \not\in E$. We can show that $E$ is an extension of $\langle D, W \rangle$. Because there is an extension $E$ such that $\psi \not\in E$, $\psi$ does not skeptically follow from $\langle D, W \rangle$. 


Proof sketch.

“⇒”: Assume there is an extension $E$ of $\langle D, W \rangle$ such that $\psi \notin E$. Hence there is an interpretation $\mathcal{I}$ such that $\mathcal{I} \models E$ and $\mathcal{I} \models \neg \psi$. By the fact that there is no extension $F$ such that $E \subset F$, $\mathcal{I}$ is a $B$-minimal model of $\Phi$. Hence $\psi$ does not $B$-minimally follow from $\Phi$.

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Relation to Default Logic: Proof

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By the fact that there is no extension \( F \) such that \( E \subset F \), \( \mathcal{I} \) is a \( B \)-minimal model of \( \Phi \). Hence \( \psi \) does not \( B \)-minimally follow from \( \Phi \).

“⇐” : Assume \( \psi \) does not \( B \)-minimally follow from \( \Phi \). Hence there is a \( B \)-minimal model \( \mathcal{I} \) of \( \Phi \) such that \( \mathcal{I} \not\models \psi \). Define

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□
Answer set semantics: a formalization of negation-as-failure in logic programming (Prolog)

Other formalizations: well-founded semantics, perfect-model semantics, inflationary semantics, ...

Can be viewed as a simpler variant of default logic

A better alternative to propositional logic in some applications
Nonmonotonic Logic Programs: Background

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Nonmonotonic Logic Programs

Let $A = \{a_1, \ldots, a_n\}$ be a set of propositions.

Rules:

$c \leftarrow b_1, \ldots, b_m, \text{not } d_1, \ldots, \text{not } d_k$

where $\{c, b_1, \ldots, b_m, d_1, \ldots, d_k\} \subseteq A$

- Meaning similar to default logic:
  - If
    1. we have derived $b_1, \ldots, b_m$ and
    2. cannot derive any of $d_1, \ldots, d_k$,
  - then derive $c$.
- Rules without right-hand side (facts): $c \leftarrow$
- Rules without left-hand side (constraints):
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Definition

Let $P$ be a set of rules without not, $\Delta \subseteq A$. The closure $\text{dcl}(P) \subseteq A$ of $P$ is defined by iterative application of the rules in the obvious way. $\Delta$ is an answer set of $P$ if $\Delta = \text{dcl}(P)$ and there is no constraint in $P$ violated by $\Delta$.

Definition (Reduct)

The reduct of a program $P$ with respect to a set of atoms $\Delta \subseteq A$ is defined as:

$$P^\Delta := \{c \leftarrow b_1, \ldots, b_m | (c \leftarrow b_1, \ldots, b_m, \text{not } d_1, \ldots, \text{not } d_k) \in P, \{d_1, \ldots, d_k\} \cap \Delta = \emptyset\}$$

Definition (Answer set)

$\Delta \subseteq A$ is an answer set of $P$ if $\Delta$ is an answer set of $P^\Delta$. 
Answer Sets – Formal Definition

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Examples

- $P_1 = \{a \leftarrow, \ b \leftarrow a, \ c \leftarrow b\}$
- $P_2 = \{a \leftarrow b, \ b \leftarrow a\}$
- $P_3 = \{p \leftarrow \text{not } p\}$
- $P_4 = \{p \leftarrow \text{not } q, \ q \leftarrow \text{not } p\}$
- $P_5 = \{p \leftarrow \text{not } q, \ q \leftarrow \text{not } p, \ \leftarrow p\}$
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Complexity: existence of answer sets is NP-complete

1  **Membership in NP:** Guess $\Delta \subseteq A$ (*nondet. polytime*), compute $P^\Delta$, compute its closure, compare to $\Delta$ (*everything det. polytime*).

2  **NP-hardness:** Reduction from 3SAT: an answer set exists iff clauses are satisfiable:

   $p \leftarrow \text{not } \hat{p}$
   $\hat{p} \leftarrow \text{not } p$

   for every proposition $p$ occurring in the clauses, and

   $\leftarrow \text{not } l'_1, \text{not } l'_2, \text{not } l'_3$

   for every clause $l_1 \lor l_2 \lor l_3$, where $l'_i = p$ if $l_i = p$ and $l'_i = \hat{p}$ if $l_i = \neg p$. 

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Programs for Reasoning with Answer Sets

- smodels (Niemelä & Simons), dlv (Eiter et al.), ...
- Schematic input:

  \begin{align*}
  p(X) & :\ - \ not \ q(X). & \quad anc(X,Y) & :\ - \ par(X,Y). \\
  q(X) & :\ - \ not \ p(X). & \quad anc(X,Y) & :\ - \ par(X,Z), \ anc(Z,Y). \\
  r(a). & & \quad par(a,b). & \quad par(a,c). \quad par(b,d). \\
  r(b). & & \quad female(a). \\
  r(c). & & \quad male(X) & :\ - \ not(female(X)). \\
  & & \quad forefather(X,Y) & :\ - \\
  & & & \quad \quad \quad \quad \quad anc(X,Y), \ male(X). \\
  \end{align*}
The *ancestor* relation is the transitive closure of the *parent* relation.

Transitive closure cannot be (concisely) represented in propositional/predicate logic.

\[
\begin{align*}
par(X, Y) & \rightarrow anc(X, Y) \\
par(X, Z) \land anc(Z, Y) & \rightarrow anc(X, Y)
\end{align*}
\]

The above formulae only guarantee that *anc* is a superset of the transitive closure of *par*.

For transitive closure one needs the minimality condition in some form: nonmonotonic logics, fixpoint logics, ...
Stratification

The reason for multiple answer sets is the fact that $a$ may depend on $b$ and simultaneously $b$ may depend on $a$. The lack of this kind of circular dependencies makes reasoning easier.

**Definition**

A logic program $P$ is **stratified** if $P$ can be partitioned to

$$P = P_1 \cup \cdots \cup P_n$$

so that for all $i \in \{1, \ldots, n\}$ and

$$(c \leftarrow b_1, \ldots, b_m, \text{not } d_1, \ldots, \text{not } d_k) \in P_i,$$

1. there is no \textbf{not }$c$ in $P_i$ and

2. there are no occurrences of $c$ anywhere in $P_1 \cup \cdots \cup P_{i-1}$. 

A stratified program $P$ has exactly one answer set. The unique answer set can be computed in polynomial time.

Example

Our earlier examples with more than one or no answer sets:

\[ P_3 = \{ p \leftarrow \text{not } p \} \]
\[ P_4 = \{ p \leftarrow \text{not } q, \quad q \leftarrow \text{not } p \} \]
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Applications of Logic Programs

1. Simple forms of default reasoning (e.g., inheritance networks, see later)

2. A solution to the frame problem: instead of using frame axioms, use defaults

\[ a_{t+1} \leftarrow a_t, \neg \neg a_{t+1} \]

By default, truth-values of facts stay the same.

3. Deductive databases (Datalog\(^\neg\))

4. Et cetera: Everything that can be done with propositional logic can also be done with propositional nonmonotonic logic programs.
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