Exercise Sheet 5
Due: Tuesday, June 8, 2010

Exercise 5.1 (Conditional independence)
This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $P(X|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

(i) $P(E_1, E_2), P(X), P(E_1|X), P(E_2|X)$
(ii) $P(E_1, E_2), P(X), P(E_1, E_2|X)$
(iii) $P(X), P(E_1|X), P(E_2|X)$

(b) Suppose we know that $P(E_1|X, E_2) = P(E_1|X)$ for all values of $X, E_1, \text{and } E_2$. Now which of the three sets are sufficient?

Exercise 5.2 (Bayesian Networks)
Consider the following Bayesian network:

(a) Rewrite the joint probability distribution $P(A, B, C, D, E, F)$ using the conditional independencies expressed by the network.

(b) Suppose that all the random variables $A, B, C, D, E, F$ in the Bayesian network can only have two possible values yes and no. What’s the minimum number of probabilities required to fully define the Bayesian network whose structure is given above?

Hint: Remember that e.g. $P(E = \text{yes}) = 1 - P(E = \text{no})$.

(c) How many probabilities would be required to define the full joint distribution over $A, B, C, D, E, F$ if we could not assume the conditional independencies expressed by the Bayesian network?

Exercise 5.3 (Bayesian Networks)
Consider the following Bayesian network:
(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network \( \text{Ind}(U, V \mid W) \) denotes that \( U \) is conditionally independent of \( V \) given \( W \), and \( \text{Ind}(U, V) \) denotes unconditional independence of \( U \) and \( V \):

(i) \( \text{Ind}(TBC, VisitToAsia) \)
(ii) \( \text{Ind}(VisitToAsia, Smoker) \)
(iii) \( \text{Ind}(VisitToAsia, PositiveXRay \mid TBCOrCancer) \)
(iv) \( \text{Ind}(VisitToAsia, Dyspnoea \mid TBCOrCancer) \)
(v) \( \text{Ind}(TBC, Smoker \mid PositiveXRay) \)

(b) Compute \( P(Dyspnoea|Smoker, \neg TBC) \). The relevant entries in the conditional probability tables are given below:

\[
\begin{align*}
P(\text{LungCancer}|\text{Smoker}) &= 0.1 \\
P(\text{LungCancer}|\neg\text{Smoker}) &= 0.01 \\
P(\text{Bronchitis}|\text{Smoker}) &= 0.2 \\
P(\text{Bronchitis}|\neg\text{Smoker}) &= 0.1 \\
P(\text{TBCOrCancer}|\text{TBC, LungCancer}) &= 1 \\
P(\text{TBCOrCancer}|\text{TBC, \negLungCancer}) &= 1 \\
P(\text{TBCOrCancer}|\neg\text{TBC, LungCancer}) &= 1 \\
P(\text{TBCOrCancer}|\neg\text{TBC, \negLungCancer}) &= 0 \\
P(\text{Dyspnoea}|\text{TBCOrCancer, Bronchitis}) &= 0.9 \\
P(\text{Dyspnoea}|\text{TBCOrCancer, \negBronchitis}) &= 0.7 \\
P(\text{Dyspnoea}|\neg\text{TBCOrCancer, Bronchitis}) &= 0.6 \\
P(\text{Dyspnoea}|\neg\text{TBCOrCancer, \negBronchitis}) &= 0.05
\end{align*}
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The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet\(^1\) and attach it to your solution.

\(^1\) http://www.informatik.uni-freiburg.de/~ki/teaching/ss10/gki/coverSheet-english.pdf