Foundations of Artificial Intelligence

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Exercise Sheet 6
Due: Friday, June 13, 2008

Exercise 6.1 (Formalization in Predicate Logic)

Let L be a binary and P und O be two unary predicates, d a constant and s a unary function. The intended semantics of the symbols is given by the interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \mathbb{N}$, $L^{\mathcal{I}} = <$, $P^{\mathcal{I}} = \{n \in \mathbb{N} \,|\, n \text{ is prime}\}$, $O^{\mathcal{I}} = \{n \in \mathbb{N} \,|\, n \text{ is odd}\}$, $d^{\mathcal{I}} = 3$ and $s^{\mathcal{I}}(n) = n + 1$ for all $n \in \mathbb{N}$. Symbolize the following statements:
 (a) Not all natural numbers are prime.
 (b) There is a prime number different from three.
 (c) For each prime number n, the number $n+1$ is not prime, unless $n+1 = 3$.
 (d) There is exactly one even prime number.
 (e) There are infinitely many prime numbers.
 (f) There is a smallest natural number.

Exercise 6.2 (Syntax and Semantics of Predicate Logic)

 (a) Classify the following expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. In the expressions, a and b are constants, x and y are variables, f and g are functions, and P and Q are predicates.

 (a) $P(x, y)$ (d) $\mathcal{I}, \alpha \models P(a, f(x))$
 (b) $f(a, b)$ (e) $f(g(x), b)$
 (c) $\mathcal{I} \models P(a, f(b))$ (f) $Q(x)$ is satisfiable.

 (g) $\exists x (P(x, y) \wedge Q(x)) \vee P(y, x)$
 (h) $\forall x (\exists y (P(x, y) \wedge Q(x)) \vee P(x, y))$
 (i) $\forall x \forall y (P(x, y) \wedge Q(x) \vee P(f(y), x))$
 (j) $Q(x) \vee P(x, y) \equiv P(x, y) \vee Q(x)$

 (b) Consider the following set of formulae:

$$\Theta = \left\{ \begin{array}{l} \forall x \neg P(x, x) \\ \forall x \forall y \forall z ((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z)) \\ \forall x \forall y (P(x, y) \vee x = y \vee P(y, x)) \end{array} \right\}$$

 Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models \Theta$ (i.e., $\mathcal{I} \models F$ for all $F \in \Theta$). Why is it not necessary to specify a variable assignment α to state a model of Θ?

 (c) Are there also models of Θ with an infinite $\mathcal{D}$?

Exercise 6.3 (Skolem Normal Form)

Transform the following formulae to Skolem normal form:
  (a) $F_1 = \forall x(\exists y R(x,y) \wedge \exists y R(y,x))$
  (b) $F_2 = \forall x \forall z(R(x,z) \Rightarrow \exists y(R(x,y) \wedge R(y,z)))$
  (c) $F_3 = \forall x \exists z(R(x,z) \wedge \neg \exists y(R(x,y) \wedge R(y,z)))$

Exercise 6.4 (Herbrand Expansion)

Let $F = \forall x \forall y(P(x, f(x, g(y))) \wedge P(h(y), f(y,y)))$.
  (a) State ten minimally large terms from the Herbrand universe of F.
  (b) State five minimally large formulae from the Herbrand expansion of F.

The exercise sheets may and should be worked on in groups of three (3) students.
Please fill the cover sheet[1] and attach it to your solution.