Best-First Search

Search procedures differ in the way they determine the next node to expand.

**Uninformed Search:** Rigid procedure with no knowledge of the cost of a given node to the goal.

**Informed Search:** Knowledge of the cost of a given node to the goal is in the form of an *evaluation function* $f$ or $h$, which assigns a real number to each node.

**Best-First Search:** Search procedure that expands the node with the "best" $f$- or $h$-value.

General Algorithm

```
function BEST-FIRST-SEARCH(problem, EVAL-FN) returns a solution sequence
    inputs: problem, a problem
            EVAL-FN, an evaluation function

    Queueing-Fn ← a function that orders nodes by EVAL-FN
    return GENERAL-SEARCH(problem, Queueing-Fn)
```

When $h$ is always correct, we do not need to search!
Greedy Search

A possible way to judge the “worth” of a node is to estimate its distance to the goal.

\[ h(n) = \text{estimated distance from } n \text{ to the goal} \]

The only real condition is that \( h(n) = 0 \) if \( n \) is a goal.

A best-first search with this function is called a greedy search.

Route-finding problem: \( h = \text{straight-line distance between two locations} \).

---

Heuristics

The evaluation function \( h \) in greedy searches is also called a heuristic function or simply a heuristic.

- The word *heuristic* is derived from the Greek word ευρίσκειν (note also: ευρηκα!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and focuses the search!
A*: Minimization of the estimated path costs

A* combines the greedy search with the uniform-search strategy.

- \( g(n) \) = actual cost from the initial state to \( n \).
- \( h(n) \) = estimated cost from \( n \) to the next goal.
- \( f(n) = g(n) + h(n) \), the estimated cost of the cheapest solution through \( n \).

Let \( h^*(n) \) be the actual cost of the optimal path from \( n \) to the next goal.

- \( h \) is admissible if the following holds for all \( n \):
  \[ h(n) \leq h^*(n) \]

We require that for A*, \( h \) is admissible (straight-line distance is admissible).

---

A* Search Example

A* Search from Arad to Bucharest

Contours in A*

Within the search space, contours arise in which for the given \( f \)-value all nodes are expanded.

Contours at \( f = 380, 400, 420 \)
Example: Path Planning for Robots in a Grid-World

Let $n$ be a node on the path from the start to $G$ that has not yet been expanded. Since $h$ is admissible, we have
\[ f(n) \leq f^*. \]
Since $n$ was not expanded before $G_2$, the following must hold:
\[ f(G_2) \leq f(n) \]
and
\[ f(G_2) \leq f^*. \]
It follows from $h(G_2) = 0$ that
\[ g(G_2) \leq f^*. \]
\[ \rightarrow \text{Contradicts the assumption!} \]

Optimality of A*

Claim: The first solution found has the minimum path cost.
Proof: Suppose there exists a goal node $G$ with optimal path cost $f^*$, but A* has found another node $G_2$ with $g(G_2) > f^*$.

Completeness and Complexity

Completeness:
If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant $\delta$ such that every operator has at least cost $\delta$.
\[ \rightarrow \text{Only a finite number of nodes } n \text{ with } f(n) \leq f^*. \]

Complexity:
In the case where $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only a sub-exponential number of nodes will be expanded.
Normally, growth is exponential because the error is proportional to the path costs.
Heuristic Function Example

\[ h_1 = \text{the number of tiles in the wrong position} \]
\[ h_2 = \text{the sum of the distances of the tiles from their goal positions (Manhatten distance)} \]

Empirical Evaluation

- \( d = \text{distance from goal} \)
- Average over 100 instances

<table>
<thead>
<tr>
<th>( d )</th>
<th>IDS</th>
<th>( A^*(h_1) )</th>
<th>( A^*(h_2) )</th>
<th>IDS</th>
<th>( A^*(h_1) )</th>
<th>( A^*(h_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
<td>6</td>
<td>2.45</td>
<td>1.79</td>
<td>1.79</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
<td>13</td>
<td>12</td>
<td>2.87</td>
<td>1.48</td>
<td>1.45</td>
</tr>
<tr>
<td>6</td>
<td>680</td>
<td>20</td>
<td>18</td>
<td>2.73</td>
<td>1.34</td>
<td>1.30</td>
</tr>
<tr>
<td>8</td>
<td>6384</td>
<td>39</td>
<td>25</td>
<td>2.80</td>
<td>1.33</td>
<td>1.24</td>
</tr>
<tr>
<td>10</td>
<td>47127</td>
<td>95</td>
<td>39</td>
<td>2.79</td>
<td>1.38</td>
<td>1.22</td>
</tr>
<tr>
<td>12</td>
<td>364404</td>
<td>227</td>
<td>73</td>
<td>2.78</td>
<td>1.42</td>
<td>1.24</td>
</tr>
<tr>
<td>14</td>
<td>3473941</td>
<td>539</td>
<td>113</td>
<td>2.83</td>
<td>1.44</td>
<td>1.23</td>
</tr>
<tr>
<td>16</td>
<td>–</td>
<td>1301</td>
<td>211</td>
<td>–</td>
<td>1.45</td>
<td>1.25</td>
</tr>
<tr>
<td>18</td>
<td>–</td>
<td>3656</td>
<td>363</td>
<td>–</td>
<td>1.46</td>
<td>1.26</td>
</tr>
<tr>
<td>20</td>
<td>–</td>
<td>7276</td>
<td>676</td>
<td>–</td>
<td>1.47</td>
<td>1.27</td>
</tr>
<tr>
<td>22</td>
<td>–</td>
<td>18094</td>
<td>1219</td>
<td>–</td>
<td>1.48</td>
<td>1.28</td>
</tr>
<tr>
<td>24</td>
<td>–</td>
<td>39135</td>
<td>1641</td>
<td>–</td>
<td>1.48</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Iterative Deepening A* Search (IDA*)

Idea: A combination of IDS and A*. All nodes inside a contour are searched.

Local Search Methods

In many problems, it is unimportant how the goal is reached – only the goal itself matters (8-queens problem, VLSI Layout, TSP).

If in addition a quality measure for states is given, a local search can be used to find solutions.

Idea: Begin with a randomly-chosen configuration and improve on it stepwise \( \rightarrow \) Hill Climbing.
Hill Climbing

```
function HILL-CLIMBING(problem) returns a solution state
  inputs: problem, a problem
  static: current, a node
    next, a node
  current ← MAKE-NODE(INITIAL-STATE(problem))
  loop do
    next ← a highest-valued successor of current
    if VALUE[next] < VALUE[current] then return current
    current ← next
  end
```

Example: 8-Queens Problem

Selects a column and moves the queen to the square with the fewest conflicts.

Problems with Local Search Methods

- **Local maxima**: The algorithm finds a sub-optimal solution.
- **Plateaus**: Here, the algorithm can only explore at random.
- **Ridges**: Similar to plateaus.

**Solutions:**

- **Start over** when no progress is being made.
- "Inject smoke" → random walk
- **Tabu search**: Do not apply the last \( n \) operators.

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.

Simulated Annealing

In the simulated annealing algorithm, "smoke" is injected systematically: first a lot, then gradually less.

```
function SIMULATED-ANNEALING(\text{problem, schedule}) returns a solution state
  inputs: problem, a problem
    schedule, a mapping from time to "temperature"
  static: current, a node
    next, a node
    \( T \), a "temperature" controlling the probability of downward steps
  current ← MAKE-NODE(INITIAL-STATE(\text{problem}))
  for \( t \leftarrow 1 \) to \( \infty \) do
    \( T \leftarrow \text{schedule}(t) \)
    if \( T \cdot \text{exp}(-\text{DECREASE}) \) return current
    next ← a randomly selected successor of current
    \text{DECREASE} ← \text{VALUE}\text{"next"} - \text{VALUE}\text{"current"}
    if \( \text{DECREASE} > 0 \) then current ← next
    else current ← next only with probability \( e^{-\text{DECREASE}} \)
```

Has been used since the early 80’s for VLSI layout and other optimization problems.
**Genetic Algorithms**

Evolution appears to be very successful at finding good solutions.

**Idea:** Similar to evolution, we search for solutions by “crossing”, “mutating”, and “selecting” successful solutions.

**Ingredients:**
- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

**Example:** 8-queens problem as a chain of 8 numbers. Fitness is judged by the number of non-attacks. The population consists of a set of arrangements of queens.

**Selection, Mutation, and Crossing**

Many variations: how selection will be applied, what type of cross-overs will be used, etc.

**Summary**

- **Heuristics** focus the search
- **Best-first search** expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal \( h \) we obtain a greedy search.
- The minimization of \( f(n) = g(n) + h(n) \) combines uniform and greedy searches. When \( h(n) \) is admissible, i.e., \( h^* \) is never overestimated, we obtain the A* search, which is complete and optimal.
- **IDA* is a combination of the iterative-deepening and A* searches.**
- **Local search methods** only ever work on one state, attempting to improve it step-wise.
- **Genetic algorithms** imitate evolution by combining good solutions.