Exercise Sheet 4
Due: Friday, May 18, 2007

Exercise 4.1 (Arc Consistency)
AC-3 puts back on the queue every arc \((X_k, X_i)\) whenever any value is deleted from the domain of \(X_i\), even if each value of \(X_k\) is consistent with several remaining values of \(X_i\). Suppose that, for every arc \((X_k, X_i)\) and each value of \(X_k\), we keep track of the number of remaining values of \(X_i\) that are consistent with this value of \(X_k\). Explain how to update these numbers efficiently and hence show that arc consistency can be enforced in total time \(O(n^2d^2)\).

Exercise 4.2 (Tree Decomposition)
You want to 3-color the following graph (say with the colors \(r, g, b\)).

\[\begin{array}{c}
A \\
B \\
C \\
D \\
E \\
F \\
G
\end{array}\]

Show a minimal tree decomposition of the graph and give the sets of all solutions for each of the subproblems. Merge the solutions of the subproblems into an overall solution in the way presented in the lecture. Write down such an overall solution.

Exercise 4.3 (\(\alpha/\beta\)-Pruning)

(a) Evaluate the following game tree using the \(\alpha/\beta\)-algorithm. For each visited interior node, give the \(\alpha\) and \(\beta\) values when the node is entered as well as the return value when exiting the node. Mark interior nodes and leaf nodes that are never visited.

(b) How do you have to reorder the moves of the players (i.e. the branches of the game tree) such that the \(\alpha/\beta\)-algorithm evaluates a minimal number of leaf nodes? Draw the reordered tree and mark the leaf nodes which are still evaluated.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.