Nondeterministic/conditional planning
Motivation

- World is not predictable.
- AI robotics:
  - imprecise movement of the robot
  - other robots
  - human beings, animals
  - machines (cars, trains, airplanes, lawn-mowers, ...)
  - natural phenomena (wind, water, snow, temperature, ...)
- Games: other players are outside our control.
- To win a game (reaching a goal state) with certainty all possible actions by the other players have to be anticipated (a winning strategy of a game).
- World is not predictable because it is unknown: we cannot observe everything.

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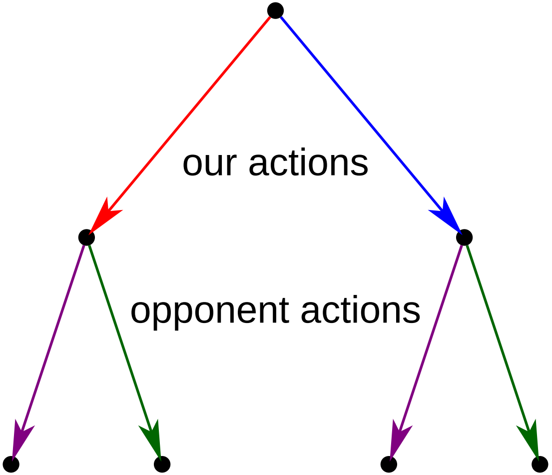
Nondeterminism

Example: several agents, games



Nondeterminism

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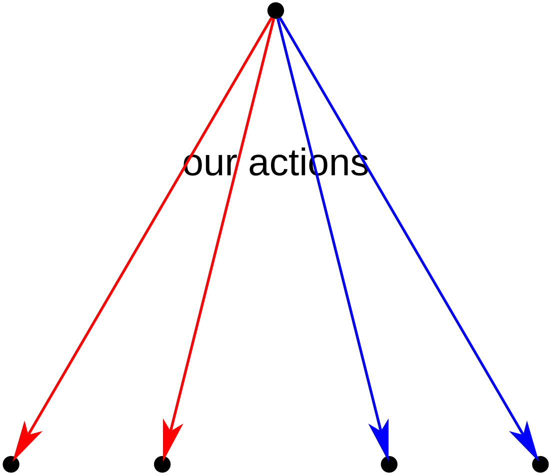
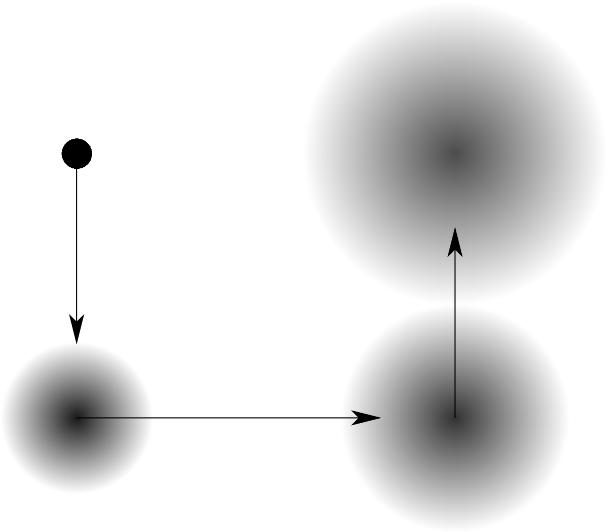


Nondeterminism

Example: uncertainty in robot movement



Nondeterministic/conditional planning
Motivation

- In deterministic planning we have assumed that the only changes taking place in the world are those caused by us and that we can exactly predict the results of our actions.
- Other agents and processes, beyond our control, are formalized as nondeterminism.
- Implications:
  1. The future state of the world cannot be predicted.
  2. We cannot reliably plan ahead: no single action sequence achieves the goals.
  3. In some cases it is not possible to achieve the goals with certainty, only with some probability.

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Transition systems

General definition with nondeterminism and observability

Definition

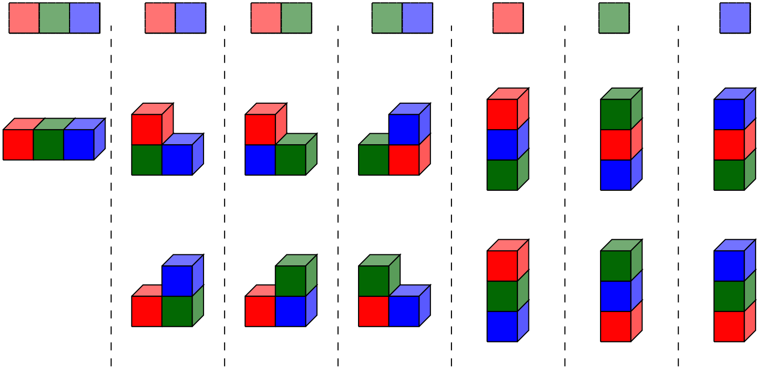
A transition system is a 5-tuple $\Pi = \langle S, I, O, G, P \rangle$ where

1. S is a finite set of states,
2. $I \subseteq S$ is the set of initial states,
3. O is a finite set of actions $o \subseteq S \times S$,
4. $G \subseteq S$ is the set of goal states, and
5. $P = (C_1, \ldots, C_n)$ is a partition of S to classes of observationally indistinguishable states satisfying $\bigcup\{C_1, \ldots, C_n\} = S$ and $C_i \cap C_j = \emptyset$ for all i, j such that $1 \leq i < j \leq n$.

Making an observation tells which set C_i the current state belongs to. Distinguishing states within a given C_i is not possible by observations.

Observability
Example of partition of states into observational classes

Blocks world with 3 blocks and camera far above the table.



Observability

Classification full, partial, no observability

Let $S = \{s_1, \dots, s_n\}$ be the set of states.

Classification of planning problems in terms of observability:

Full $P = (\{s_1\}, \{s_2\}, \dots, \{s_n\})$
number of observational classes: n
Chess is a fully observable 2-person game.

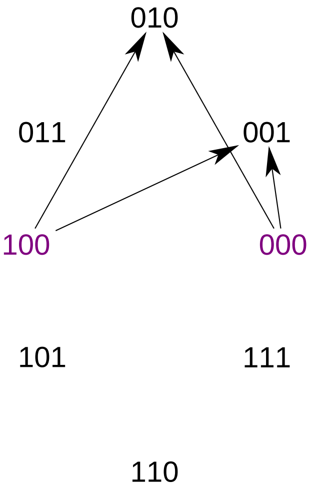
No $P = (\{s_1, \dots, s_n\})$
number of observational classes: 1

Partial No restrictions on P.
number of observational classes : between 1 and n
Poker is a partially observable 2-person game.
Mastermind is a partially observable 1-person game.

n-person games for $n \geq 2 \sim$ nondeterministic planning

Nondeterministic actions as operators

Example



	000	001	010	011	100	101	110	111
000	0	1	1	0	0	0	0	0
001	0	0	0	0	0	0	0	0
010	0	0	0	0	0	0	0	0
011	0	0	0	0	0	0	0	0
100	0	1	1	0	0	0	0	0
101	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0

In terms of state variables $A = \{a, b, c\}$ the action can be represented as operator

$$\langle \neg b \land \neg c, \neg a \land (b|c) \rangle$$

Nondeterministic actions as operators
Definition

Definition

Let A be a set of state variables. An operator is a pair $\langle c, e \rangle$ where c is a propositional formula over A (the precondition), and e is an effect over A. Effects over A are recursively defined as follows.

1. a and $\neg a$ for state variables $a \in A$ are effects over A.
2. $e_1 \wedge \cdots \wedge e_n$ is an effect over A if $e_1, \ldots, e_n$ are effects over A.
3. $c \rhd e$ is an effect over A if c is a formula over A and e is an effect over A.
4. $e_1 | \cdots | e_n$ is an effect over A if $e_1, \ldots, e_n$ for $n \geq 2$ are effects over A.

Nondeterministic operators

Semantics, example

Example

$$\langle a, (b|\neg b) \land (c|\neg c) \land (d|\neg d)\rangle$$

has 2^3 alternative sets of effects, leading to 8 different successor states.

1. effects $\{b, c, d\}$ lead to state $s \models a \land b \land c \land d$
2. effects $\{\neg b, c, d\}$ lead to state $s \models a \land \neg b \land c \land d$
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Nondeterministic operators
Semantics

Definition (Operator application)

Let $\langle c, e \rangle$ be an operator over A and s a state.
The set $[e]_s$ of sets of literals is recursively defined as follows.

1. $[a]_s = \{\{a\}\}$ and $[\neg a]_s = \{\{\neg a\}\}$ for $a \in A$.
2. $[e_1 \wedge \cdots \wedge e_n]_s = \{\bigcup_{i=1}^{n} E_i | E_1 \in [e_1]_s, \ldots, E_n \in [e_n]_s\}$.
3. $[c' \rhd e]_s = [e]_s$ if $s \models c'$ and $[c' \rhd e]_s = \{\emptyset\}$ otherwise.
4. $[e_1 | \cdots | e_n]_s = [e_1]_s \cup \cdots \cup [e_n]_s$.

Definition

Operator $\langle c, e \rangle$ is applicable in s if $s \models c$ and every set $E \in [e]_s$ is consistent.

Nondeterministic operators
Binary relation induced by an operator

Definition

An operator $\langle c, e\rangle$ induces a binary relation $R\langle c, e\rangle$ on the states as follows: $sR\langle c, e\rangle s'$ if there is $E \in [e]_s$ such that

1. $s \models c$,
2. $s' \models E$, and
3. $s \models a$ iff $s' \models a$ for all $a \in A$ such that $\{a, \neg a\} \cap E = \emptyset$.

We also write simply sos' instead of $sR(o)s'$.

Definition

Let s and s' be states and o an operator. If sos' then s' is a successor state of s.

Succinct transition systems
General definition

Definition

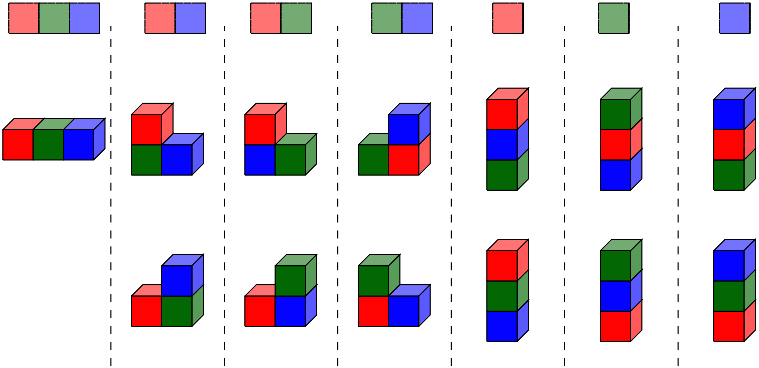
A succinct transition system is a 5-tuple $\Pi = \langle A, I, O, G, V \rangle$ where

1. A is a finite set of state variables,
2. I is a formula over A describing the initial states,
3. O is a finite set of operators over A,
4. G is a formula over A describing the goal states, and
5. $V \subseteq A$ is the set of observable state variables.

Observability

Example of partition of states into observational classes

State variables $V = \{Aclear, Bclear, Cclear\}$ are observable.



There are 8 valuations of V but the valuation $v \models \neg Aclear \wedge \neg Bclear \wedge \neg Cclear$ does not correspond to a blocks world state.

Succinct transition systems
Observability

Let $A = \{a_1, \dots, a_n\}$ be the state variables.
Classification of planning problems in terms of observability:

Full observable state variables: $V = A$
number of observational classes: $2^{|A|}$

No observable state variables: $V = \emptyset$
number of observational classes: 1

Partial observable state variables: no restrictions, $\emptyset \subseteq V \subseteq A$
number of observational classes: 1 to $2^{|A|}$

We can associate a transition system with every succinct transition system.

Definition

Given a succinct transition system $\Pi = \langle A, I, O, G, V \rangle$, construct the transition system $F(\Pi) = \langle S, I', O', G', P \rangle$ where

1. S is the set of all Boolean valuations of A,
2. $I' = \{s \in S | s \models I\}$,
3. $O' = \{R(o) | o \in O\}$,
4. $G' = \{s \in S | s \models G\}$, and
5. $P = (C_1, \ldots, C_n)$ where $v_1, \ldots, v_n$ for $n = 2^{|V|}$ are all the Boolean valuations of V and $C_i = \{s \in S | s(a) = v_i(a) \text{ for all } a \in V\}$ for all $i \in \{1, \ldots, n\}$.