	Invariants Motivation			
Invariants (May 9, 2005)	Invariants			
Invariants (May 9, 2005)	Motivation			
Invariants Motivation				
Definition	Example			
Example	Consider the goal formula			
vs. Plan existence	AonB ∧ BonC			
Algorithms Idea	regressed with operator			
Example				
Invariant test	$\langle AonC \land Aclear \land Bclear, AonB \land \neg Bclear \land Cclear \rangle$			
Main procedure	giving new goal			
Example Applications	AonC ∧ Aclear ∧ Bclear ∧ BonC.			
SAT Planning	AUTO A Aclear A Bulear A Bullo.			
Regression	It is intuitively clear that no state satisfying this formula is reachable by			
Summary	any plan from a legal blocks world state.			
(Albert-Ludwigs-Universität Freiburg) 1 / 28	(Albert-Ludwigs-Universität Freiburg) Al Planning May 9, 2005 2 / 28			
Invariants Motivation	Invariants Motivation			
Invariants	Invariants			
Motivation				
Goal formulae and formulae obtained by regression from them often represent some states that are not reachable from the initial	Goal: Restriction to states that are reachable.			
state.	Problem: Testing reachability is computationally as complex as			
If none of the states is reachable from the initial state because	testing whether a plan exists.			
there are no plans reaching the formula.	Solution: Use an approximate notion of reachability.			
We would like to have reachable states only, if possible.	Implementation: Compute in polynomial time formulae that			
Same problem shows up in satisfiability planning: partial	characterize a superset of the reachable states.			
valuations considered by satisfiability algorithms may represent unreachable states, and this may result in unnecessary search.				
(Albert-Ludwigs-Universität Freiburg) Al Planning May 9, 2005 3 / 28	(Albert-Ludwigs-Universität Freiburg) Al Planning May 9, 2005 4 / 28			
Invariants Definition	Invariants Definition			
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Invariants: definition				
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Invariants vs. Plan existence

Invariants: connection to plan existence

Proof continues. Let $o = \langle q, a_1 \land \cdots \land a_n \rangle$ with $A = \{a_1, \ldots, a_n, q\}$. For $\langle A, I, O, q \rangle$ a plan exists iff for $\langle A, I, O \cup \{o\}, q \rangle$ a plan exists

iff for $\langle A, I, O \cup \{o\}, q \land a_1 \land \cdots \land a_n \rangle$ a plan exists. Testing satisfiability of $\phi \land q \land a_1 \land \cdots \land a_n$ can be done in polynomial time: replace every state variable in the strongest invariant ϕ by \top and

simplify, getting \top or \bot . So, if we had a polynomial-time algorithm for computing the strongest invariant ϕ , we could test plan existence in polynomial time.

Hence plan existence is polynomial-time reducible to computing the strongest invariant.

Since the former is PSPACE-hard also the latter is PSPACE-hard.

(Albert-Ludwigs-Universität Freiburg)	Al Planning	May 9, 2005 9 / 28
	Algorithms Idea	

Computation of invariants: informally

- 1. Start with all 1-literal clauses that are true in the initial state.
- 2. Repeatedly test every operator vs. every clause, whether the clause can be shown to be true after applying the operator:
 - 2.1 One of the literals in the clause is necessarily true: retain.
 - 2.2 Otherwise, if the clause is too long: forget it.
 - 2.3 Otherwise, replace the clause by new clauses obtained by adding literals that are now true.
- 3. When all clauses remain, stop: they are invariants.

Algorithms Example

Al Planning

Computation of invariants Example

Example continues..

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- 7. For \neg *Bclear* and \neg *AonT* we respectively get
 - \neg Bclear \lor Aclear, \neg Bclear \lor Bclear, \neg Bclear $\lor \neg$ AonB, \neg Bclear \lor
 - \neg BonA, \neg Bclear \lor AonT, \neg Bclear \lor BonT and
 - \neg Aon $T \lor$ Aclear, \neg Aon $T \lor$ Bclear, \neg Aon $T \lor \neg$ AonB, \neg Aon $T \lor$
 - \neg BonA, \neg AonT \lor AonT, \neg AonT \lor BonT.
- 8. By eliminating logically equivalent ones, tautologies, and those that follow from those in C_0 not falsified we get $C_1 = \{ Aclear, \neg BonA, BonT, AonB \lor Bclear, AonB \lor AonT, \neg Bclear \lor$ \neg *AonB*, \neg *Bclear* \lor *AonT*, \neg *AonT* \lor *Bclear*, \neg *AonT* \lor \neg *AonB*} for
- distance 1 states. 9. The precondition of

 $\langle Bclear \land BonT \land Aclear, BonA \land \neg Aclear \land \neg BonT \rangle$ is satisfiable with C_1 , and the set C_2 contains all invariants for 2 blocks. AI Planning

(Albert-Ludwigs-Universität Freiburg)

Algorithms Invariant test

AI Planning

Computation of invariants: procedure preserved Test whether a clause remains true when operator is applied

PROCEDURE preserved($l_1 \lor \cdots \lor l_n, C, o$); $\langle c, e \rangle := o;$ FOR EACH $l \in \{l_1, \dots, l_n\}$ DO $IF C \cup \{EPC_{\overline{l}}(o)\} \models l' \land \neg EPC_{\overline{l'}}(e) THEN GOTO OK;$ END DO RETURN false; OK: END DO RETURN true:

Computation of invariants: informally

Similar to distance estimation with D_i^{max} : compute sets C_i of *n*-literal clauses characterizing (giving an upper bound!) the states that are reachable in *i* steps.

Example

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C_0 = \{a, \neg b, c\} \sim \{101\}
                                                                     a, \neg b falsified
C_1 = \{a \lor b, \neg a \lor \neg b, c\} \sim \{101, 011\}
C_2 = \{\neg a \lor \neg b, c\} \sim \{001, 011, 101\}
                                                                    a \vee b falsified
C_3 = \{\neg a \lor \neg b, c \lor a\} \sim \{001, 011, 100, 101\} c falsified
C_4 = \{\neg a \lor \neg b\} \sim \{000, 001, 010, 011, 100, 101\} \ c \lor a \text{ falsified}
C_5 = \{\neg a \lor \neg b\} \sim \{000, 001, 010, 011, 100, 101\}
C_i = C_5 for all i > 5
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Al Planning

May 9, 2005 10 / 28

May 9, 2005 12 / 28

May 9, 2005 14 / 28

 $\neg a \lor \neg b$ is the only invariant found.

Algorithms Example

Computation of invariants Example

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Example

Let $C_0 = \{ Aclear, \neg Bclear, AonB, \neg BonA, \neg AonT, BonT \}$ and $o = \langle \text{Aclear} \land \text{AonB}, \text{Bclear} \land \neg \text{AonB} \land \text{AonT} \rangle.$

- 1. $C_0 \cup \{Aclear \land AonB\}$ is satisfiable: *o* is applicable.
- 2. The 1-literal clauses ¬Bclear, AonB and ¬AonT become false when o is applied.
- 3. They are not thrown away, like we did when computing D_i^{max} . They are replaced by weaker clauses.
- 4. Literals true after applying *o* in state *s* such that $s \models C$: Aclear, Bclear, ¬AonB, ¬BonA, AonT, BonT
- 5. 2-literal clauses that are weaker than AonB and now true are Aon $B \lor$ Aclear, Aon $B \lor$ Bclear, Aon $B \lor \neg$ AonB, Aon $B \lor$ \neg BonA, AonB \lor AonT, AonB \lor BonT.

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Example

- Let $C_i = \{\neg AinRome \lor \neg AinNYC, \neg AinParis \lor AinParis \lor \neg AinParis \lor AinParis$ \neg AinParis $\lor \neg$ AinNYC},
 - $o = \langle AinRome, AinParis \land \neg AinRome \rangle.$
- 1. Does *o* preserve truth of $\neg AinParis \lor \neg AinNYC?$
- 2. Because o makes $\neg AinParis$ false, we must show that $\neg AinNYC$ is true after applying o.
- 3. But $\neg AinNYC$ is not even mentioned in o!
- 4. However, since AinRome is the precondition of o and \neg *AinRome* $\lor \neg$ *AinNYC* was true before applying *o*, we can infer that ¬AinNYC was true before applying o.
- 5. Since *o* does not make ¬*AinNYC* false, it is true also after applying o, and then so is $\neg AinParis \lor \neg AinNYC$.

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Computation of invariants: function preserved

Let $C = \{c \lor b\}.$

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- 1. preserved($a \lor b$, C, $\langle \neg c, c \land d \rangle$) returns *true*
- 2. preserved($a \lor b$, C, $\langle \neg c, \neg a \land b \rangle$) returns *true*
- 3. preserved($a \lor b$, C, $\langle b, \neg a \rangle$) returns *true*
- 4. preserved($a \lor b$, C, $\langle \neg c, \neg a \rangle$) returns *true*
- 5. preserved($a \lor b$, C, $\langle c, \neg a \rangle$) returns false

May 9, 2005 13 / 28

May 9, 2005 11 / 28

Let *C* be a set of clauses, $\phi = l_1 \vee \cdots \vee l_n$ a clause, and *o* an operator.

If preserved(ϕ ,C,o) returns true, then app_o(s) $\models \phi$ for every state s

Computation of invariants: function preserved Correctness

Algorithms Invariant test

Computation of invariants: function preserved Why is preserved incomplete?

Example

Let $o = \langle a, \neg b \land (c \rhd d) \land (\neg c \rhd e) \rangle$. preserved $(b \lor d \lor e, \emptyset, o)$ returns false because it cannot prove for any literal in $b \lor d \lor e$ that it is true after application of o. pplying o, and hence $b \lor d \lor e$ will be true

 $app_{o_1;...;o_{i-1}}(I) \models \phi$. Since $\phi \in C_i$ preserved(ϕ, C_{i-1}, o) returns

2. As $\phi' \in C_{i-1}$ by induction hypothesis $app_{o_1,...,o_{i-1}}(I) \models \phi'$. 3. Since $\phi' \models \phi$ also $app_{o_1:...;o_{i-1}}(I) \models \phi$. 4. Since preserved (ϕ, C_i, ϕ) returns *true* $app_{o_1:...;o_i}(I) \models \phi$ by the

1. As $\phi \notin C_{i-1}$ there is $\phi' \in C_{i-1}$ with $\phi = \phi' \vee l'_1 \vee \cdots \vee l'_m$ for some

 U'_1, \dots, U'_m and preserved (ϕ', C_{i-1}, o') returns *false* for some $o' \in O$. Hence $\phi' \models \phi$.

Inductive case $i \ge 1$: Take any $\{o_1, \ldots, o_i\} \subseteq O$ and $\phi \in C_i$.

A Consider the case $\phi \in C_{i-1}$. By induction hypothesis

true. Hence by the Lemma $app_{o_1;...;o_i}(I) \models \phi$.

such that $s \models C$ and $app_o(s)$ is defined.			literal in $b \lor d \lor e$ that it is true after application of o . However, $d \lor e$ is true after applying o , and hence $b \lor d \lor e$ will as well.			
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	Algorithms Main procedure		Algorithms Main procedure			
Computation of inva	Computation of invariants: the main procedure		Computation of invariants: the main procedure			
 C = the set of 1-literal clauses that are true in the initial state. For each operator <i>o</i> and clause l₁ ∨ ··· ∨ l_m ∈ C test if l₁ ∨ ··· ∨ l_m remains true when <i>o</i> is applied. If not, remove l₁ ∨ ··· ∨ l_m, and if m < n add clauses l₁ ∨ ··· ∨ l_m ∨ a and l₁ ∨ ··· ∨ l_m ∨ ¬a for every a ∈ A. Repeat from step 2 if C has changed. Otherwise every clause in C is an invariant. The number of iterations is O(mⁿ) which is polynomial in the number of state variables m = A for any fixed n. 			$\begin{array}{l} \label{eq:proceeding} \textit{PROCEDURE invariants}(A, I, O, n);\\ C := \{a \in A I \models a\} \cup \{\neg a a \in A, I \not\models a\};\\ \textit{REPEAT}\\ C' := C;\\ \textit{FOR EACH } l_1 \lor \cdots \lor l_m \in C \textit{ AND } o \in O\\ \textit{ such that preserved}(l_1 \lor \cdots \lor l_m, C', o) = \textit{false DO}\\ C := C \setminus \{l_1 \lor \cdots \lor l_m\};\\ \textit{IF } m < n \textit{ THEN}\\ C := C \cup \bigcup_{a \in A} \{l_1 \lor \cdots \lor l_m \lor a, \ l_1 \lor \cdots \lor l_m \lor \neg a\};\\ \textit{END FOR}\\ \textit{UNTLC } C = C';\\ \textit{RETURN } C; \end{array}$			
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	Algorithms Main procedure			Algorithms Main procedure		
Computation of invariants: the main procedure			Computation of invariants: the main procedure			

Theorem

Lemma

The procedure invariants (A, I, O, n) returns a set C of clauses with at most n literals so that for any sequence o_1, \ldots, o_m of operators in O $app_{o_1;\ldots;o_m}(I) \models C.$

Proof.

Let C_0 be the value first assigned to the variable C and C_1, C_2, \ldots the values of C in the end of each iteration.

Induction hypothesis: for every $\{o_1, \ldots, o_i\} \subseteq O$ and $\phi \in C_i$, $app_{o_1;\ldots;o_i}(I) \models \phi.$

Base case i = 0: $app_{e}(I)$ for the empty sequence is by definition I itself, and by construction C₀ consists of only formulae that are true in the initial state.

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Algorithms Main procedure

Why is the strongest invariant not always found?

- 1. Practical implementations of the algorithm use polynomial time approximations of the tests for satisfiability and \models .
- 2. The function preserved is incomplete for operators in general (but complete for STRIPS operators.) Making it complete makes it NP-hard.
- 3. The strongest invariant may require arbitrarily long clauses, so the restriction to clauses of any fixed length makes it impossible to represent it.

Example

The acyclicity of the on relation in the blocks world needs clauses of length n when there are n blocks.

Al Planning

May 9, 2005 18 / 28

May 9, 2005 20 / 28

22/28

May 9, 2005

Algorithms Example

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Computation of invariants Example

Lemma.

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B Consider the case $\phi \notin C_{i-1}$.

Proof continues.

Initial state: $I \models a \land \neg b \land \neg c$ **Operators**: $o_1 = \langle a, \neg a \land b \rangle$, $o_2 = \langle b, \neg b \wedge c \rangle,$ $o_3 = \langle c, \neg c \land a \rangle$

Computation: Find invariants with at most 2 literals:

 $C_0 = \{\mathbf{a}, \neg b, \neg c\}$ $C_1 = \{\neg \mathbf{c}, \mathbf{a} \lor \mathbf{b}, \neg \mathbf{b} \lor \neg \mathbf{a}\}$ $C_2 = \{\neg b \lor \neg a, \neg \mathbf{c} \lor \neg \mathbf{a}, \neg \mathbf{c} \lor \neg \mathbf{b}\}$ $C_3 = \{\neg b \lor \neg a, \neg c \lor \neg a, \neg c \lor \neg b\}$ $C_j = C_2$ for all $j \ge 2$

Applications SAT Planning

Invariants in satisfiability planning

Invariants in satisfiability planning

For every invariant $l_1 \vee \cdots \vee l_n$ add the clauses

 $l_1^t \lor \cdots \lor l_n^t$

for all time points t.

Notice that the above formulae logical consequences of Φ_i^{seq} and Φ_i^{par} , so the invariants do not change the set of valuations of these formulae.

Invariants are critical for the efficiency of satisfiability planning on many types of problems.

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Applications Regression Invariants in backward search Motivating example			Summary			
 Problem: Regression produces sets <i>T</i> of states such that 1. some states in <i>T</i> are not reachable from <i>I</i>, or 2. none of the states in <i>T</i> are reachable from <i>I</i>. The first is not always a serious problem (but may worsen the quality of distance estimates, for example.) Solution: Use invariants to avoid formulae that do not represent any 		 Invariants are needed for making backward search and satisfiability planning more efficient. We gave an algorithm for computing a class of invariants. 1. Start with 1-literal clauses true in the initial state. 2. Repeatedly weaken clauses that could not be shown to be invariants. 3. Stop when all clauses are guaranteed to be invariants. 				
reachable states.		 The algorithm runs in polynomial time if the satisfiability and 				

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- 1. Compute invariant ϕ .
- 2. Do only regression steps such that $regr_o(\psi) \wedge \phi$ is satisfiable.

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May 9, 2005 27 / 28

Invariants in backward search

Motivating example

Example Regression of in(A,Freiburg) by (in(A,Strassburg), ¬in(A,Strassburg)∧in(A,Paris)) gives in(A,Freiburg) \in(A,Strassburg) No state satisfying in(A,Freiburg) \wedge in(A,Strassburg) makes sense if A denotes some usual physical object.

Applications Regression

logical consequence tests are approximated by a polynomial time algorithm.

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May 9, 2005 28 / 28