Transition systems (April 13, 2005)

Transition systems Definition Example Matrices Reachability Algorithm

Succinct transition systems State variables Propositional logic Operators

Schematic operators

(Albert-Ludwigs-Universität Freiburg)

Transition systems

(Albert-Ludwigs-Universität Freiburg)

Transition systems

completely predictable.

▶ $I \in S$ is a state,

Definition

Deterministic transition systems



Al Planning

A transition system is deterministic if there is only one initial state and all actions are deterministic. Hence all future states of the world are

Transition systems

A deterministic transition system is $\langle S, I, O, G \rangle$ where

► S is a finite set of states (the state space),

Definitio

Transition systems

April 13, 2005 2 / 60

April 13, 2005 4 / 60

Definition Transition systems Transition systems Formalization of the dynamics of the world/application

Definition

A transition system is $\langle S, I, \{a_1, \ldots, a_n\}, G \rangle$ where

- ► S is a finite set of states (the state space),
- $I \subseteq S$ is a finite set of initial states,
- every action $a_i \subseteq S \times S$ is a binary relation on S,
- $G \subseteq S$ is a finite set of goal states.

Definition

An action a is applicable in a state s if sas' for at least one state s'.

(Albert-Ludwigs-Universität Freiburg) Al Planning April 13, 2005 3 / 60 Transition systems Example Blocks world The rules of the game Location on the table does not matter Ξ Location on a block does not matter



At most one block on/under a block is allowed



April 13, 2005 5 / 60

1/60

Example on systems

Blocks world Properties

blocks	states
1	1
2	3
3	13
4	73
5	501
6	4051
7	37633
8	394353
9	4596553

- 10 58941091
- 1. Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration)
- 2. Finding a shortest solution is NP-complete (for a compact description of the problem).

AI Planning

(Albert-Ludwigs-Universität Freiburg)

G

6/60

▶ actions $a \in O$ (with $a \subseteq S \times S$) are partial functions, • $G \subseteq S$ is a finite set of goal states. Successor state wrt. an action Given a state s and an action A so that a is applicable in s, the successor state of s with respect to a is s' such that sas', denoted by $s' = app_a(s).$ (Albert-Ludwigs-Universität Freiburg) Al Planning Transition systems Example Blocks world The transition graph for three blocks



Transition systems Example

Deterministic planning: plans

Definition

A plan for $\langle S, I, O, G \rangle$ is a sequence $\pi = o_1, \ldots, o_n$ of operators such that $o_1, \ldots, o_n \in O$ and s_0, \ldots, s_n is a sequence of states (the execution of π) so that

1.
$$s_0 = I$$
,

2. $s_i = app_{o_i}(s_{i-1})$ for every $i \in \{1, ..., n\}$, and

3. $s_n \in G$.

$$\mathsf{app}_{o_n}(\mathsf{app}_{o_{n-1}}(\cdots \mathsf{app}_{o_1}(I)\cdots)) \in$$

Transition systems Matrices

Transition relations as matrices

- If there are *n* states, each action (a binary relation) corresponds to an *n* × *n* matrix: Element at row *i* and column *j* is 1 if the action maps state *i* to state *j*, and 0 otherwise.
 For deterministic actions there is at most one non-zero element in each row.
- 2. Matrix multiplication corresponds to sequential composition: taking action M_1 followed by action M_2 is the product M_1M_2 . (This also corresponds to the join of the relations.)
- 3. The unit matrix $I_{n\times n}$ is the NO-OP action: every state is mapped to itself.



Sum matrix $M_R + M_G + M_B$ Representing one-step reachability by any of the component actions

Transition systems Matrices



We use addition 0 + 0 = 0 and b + b' = 1 if b = 1 or b' = 1.



Al Planning

April 13, 2005 13 / 60

Transition systems Reachability

Reachability

Let M be the $n\times n$ matrix that is the (Boolean) sum of the matrices of the individual actions. Define

$$\begin{aligned} R_0 &= I_{n \times n} \\ R_1 &= I_{n \times n} + M \\ R_2 &= I_{n \times n} + M + M^2 \\ R_3 &= I_{n \times n} + M + M^2 + M^3 \\ \vdots \end{aligned}$$

 R_i represents reachability by *i* actions or less. If s' is reachable from s, then it is reachable with $\leq n - 1$ actions: $R_{n-1} = R_n$.

Al Planning

Example



Transition systems Matrices







Al Planning

Transition systems

Transition systems Matrices

Sequential composition as matrix multiplication

(01000)	(010	0 0	0 \		(000001	Ι
000001	000	0 0	1		100010	
011001	011	0 0	1		111011	
101010	101	0 1	0	-	011100	
010100	010	1 0	0		001011	
\100010/	\100	0	0/		(010100)	Ϊ

E is reachable from B by two actions because

F is reachable from B by one action and E is reachable from F by one action.

(Albert-Ludwigs-Universität Freiburg)

Al Planning

April 13, 2005 14 / 60

April 13, 2005 10 / 60



Reachability: example, M_R



Al Planning

Transition systems Reachability Reachability: example, $M_R + M_R^2$

|A B C D E FA 0 1 0 0 0 1 D B 0 0 0 0 1 1 C 0 0 1 0 0 0 $D \mid 0 \ 0 \ 1 \ 0 \ 0 \ 0$ E E 0 1 0 0 0 1 F 0 1 0 0 1 0

Transition systems Reachability Reachability: example, $M_R + M_R^2 + M_R^3 + I_{6\times 6}$

AI Planning

April 13, 2005 17 / 60

(Albert-Ludwigs-Universität Freiburg)



A simple planning algorithm

- We next present a simple planning algorithm based on computing distances in the transition graph.
- > The algorithm finds shortest paths less efficiently than Dijkstra's algorithm; we present the algorithm because we later will use it as a basis of an algorithm that is applicable to much bigger state spaces than Dijkstra's algorithm directly.

(Albert-Ludwigs-Universität Freiburg)	AI Planning	April 13, 2005	21 / 60		
	Transition systems Algorithm				
A simple planning algorithm					

- 1. Compute the matrices $R_0, R_1, R_2, \ldots, R_n$ representing reachability with $0, 1, 2, \ldots, n$ steps with all actions.
- 2. Find the smallest i such that a goal state \boldsymbol{s}_g is reachable from the initial state according to R_i .
- 3. Find an action (the last action of the plan) by which s_g is reached with one step from a state $s_{g'}$ that is reachable from the initial state according to R_{i-1} .

Al Planning

4. Repeat the last step, now viewing $s_{g'}$ as the goal state with distance i - 1.

Reachability: example, $M_R + M_R^2 + M_R^3$

Transition systems Reachability



(Albert-Ludwigs-Universität Freiburg)

April 13, 2005 18 / 60

Al Planning

Relations and sets as matrices Row vectors as sets of states

Row vectors S represent sets. SM is the set of states reachable from S by M.

(1)	Т	/1	1	0	0	1	1 \		(1)	Т
0		0	1	0	0	1	1		1	
1		0	0	1	0	0	0		1	
0	×	0	0	1	1	0	0	=	0	
0		0	1	0	0	1	1		1	
\o/		0/	1	0	0	1	1/		(1)	

Al Planning

Transition systems Algorithm

A simple planning algorithm

(Albert-Ludwigs-Universität Freiburg)



(Albert-Ludwigs-Universität Freiburg)

Al Planning

April 13, 2005 22 / 60

April 13, 2005 20 / 60

Transition systems

Example



 $D \mid 0 \mid 0 \mid 0 \mid 0$

D 0 0 0 0

D 0 0 0 0



- 4. If ϕ and ϕ' are propositional formulae, then so is $\phi \wedge \phi'$.
- 5. The symbols \perp and \top are propositional formulae.
- The implication $\phi \rightarrow \phi'$ is an abbreviation for $\neg \phi \lor \phi'$.
- The equivalence $\phi \leftrightarrow \phi'$ is an abbreviation for $(\phi \rightarrow \phi') \land (\phi' \rightarrow \phi)$.

Al Planning

5. $v \models \top$

6. $v \not\models \bot$

3. $v \models \phi \lor \phi'$ if and only if $v \models \phi$ or $v \models \phi'$

4. $v \models \phi \land \phi'$ if and only if $v \models \phi$ and $v \models \phi'$

Succinct transition systems Propositional logic

Propositional logic

Some terminology

- A propositional formula φ is satisfiable if there is at least one valuation v so that v ⊨ φ. Otherwise it is unsatisfiable.
- A propositional formula φ is valid or a tautology if v ⊨ φ for all valuations v. We write this as ⊨ φ.
- A propositional formula φ is a logical consequence of a propositional formula φ', written φ' ⊨ φ, if v ⊨ φ for all valuations v such that v ⊨ φ'.
- ▶ A propositional formula that is a proposition a or a negated proposition $\neg a$ for some $a \in A$ is a literal.
- A formula that is a disjunction of literals is a clause.

(Albert-Ludwigs-Universität Freiburg)	Al Planning	April 13, 2005
Succ	inct transition systems Operators	
Operators		

Succinct transition systems Propositional logi

Formulae vs. sets

sets	formulae
those $\frac{2^n}{2}$ states in which <i>a</i> is true	$a \in A$
$E \cup F$	$E \lor F$
$E \cap F$	$E \wedge F$
$E \setminus F$ (set difference)	$E \wedge \neg F$
\overline{E} (complement)	$\neg E$
the empty set \emptyset the universal set	⊥ ⊤
question about sets	question about formulae
$E \subseteq F$?	$E \models F$?
$E \subset F$?	$E \models F$ and $F \not\models E$?
E = F?	$E \models F$ and $F \models E$?

Al Planning

(Albert-Ludwigs-Universität Freiburg)

April 13, 2005 34 / 60

April 13, 2005 36 / 60

Succinct transition systems Operators

Effects

For deterministic operators

Actions are represented as operators $\langle c, e \rangle$ where

- c (the precondition) is a propositional formula over A describing the set of states in which the action can be taken. (States in which an arrow starts.)
- e (the effect) describes the successor states of states in which the action can be taken. (*Where do the arrows go.*)
 The description is procedural: how do the values of the state variable change?

Al Planning

 $c \triangleright e$ means that change *e* takes place if *c* is true in the current state.

Succinct transition systems Operators

Definition

Effects are then recursively defined as follows.

- 1. a and $\neg a$ for state variables $a \in A$ are effects.
- e₁ ∧ · · · ∧ e_n is an effect if e₁, . . . , e_n are effects (the special case with n = 0 is the empty conjunction ⊤.)

Al Planning

3. $c \triangleright e$ is an effect if c is a formula and e is an effect.

Atomic effects a and $\neg a$ are best understood respectively as assignments a := 1 and a := 0.

Succinct transition systems Operators

(Albert-Ludwigs-Universität Freiburg)

Example: operators for blocks world

For convenience we use also state variables *Aclear*, *Bclear*, and *Cclear* to denote that there is nothing on the block in question.

 $\begin{array}{l} \langle \textit{Aclear} \land \textit{AonT} \land \textit{Bclear}, \quad \textit{AonB} \land \neg \textit{AonT} \land \neg \textit{Bclear} \rangle \\ \langle \textit{Aclear} \land \textit{AonT} \land \textit{Cclear}, \quad \textit{AonC} \land \neg \textit{AonT} \land \neg \textit{Cclear} \rangle \end{array}$

 $\begin{array}{l} \langle Aclear \wedge AonB, \ AonT \wedge \neg AonB \wedge \neg AonC \rangle \\ \langle Aclear \wedge AonC, \ AonT \wedge \neg AonB \wedge \neg AonC \rangle \\ \langle Bclear \wedge BonA, \ BonT \wedge \neg BonA \wedge Aclear \rangle \\ \langle Bclear \wedge BonC, \ BonT \wedge \neg BonC \wedge Cclear \rangle \end{array}$

(Albert-Ludwigs-Universität Freiburg)

April 13, 2005 38 / 60

Succinct transition systems Operators

Operators: the successor state of a state

Definition (Successor state) The successor state $app_o(s)$ of s with respect to operator $o = \langle c, e \rangle$ is obtained from s by making literals in $[e]_s$ true. This is defined only if o is applicable in s.

Al Planning

Example Consider the operator $\langle a, \neg a \land (\neg c \rhd \neg b) \rangle$ and a state *s* such that $s \models a \land b \land c$. The operator is applicable because $s \models a$ and $[\neg a \land (\neg c \rhd \neg b)]_s = \{\neg a\}$ is consistent. Hence $app_{\langle a, \neg a \land (\neg c \rhd \neg b) \rangle}(s) \models \neg a \land b \land c$.

Operators: meaning

Changes caused by an operator

Assign each effect e and state s a set $[e]_s$ of literals as follows.

Succinct transition systems Operators

- 1. $[a]_s = \{a\}$ and $[\neg a]_s = \{\neg a\}$ for $a \in A$.
- 2. $[e_1 \wedge \cdots \wedge e_n]_s = [e_1]_s \cup \ldots \cup [e_n]_s.$
- 3. $[c \triangleright e]_s = [e]_s$ if $s \models c$ and $[c \triangleright e]_s = \emptyset$ otherwise.

Applicability of an operator

Operator $\langle c, e \rangle$ is applicable in a state *s* iff $s \models c$ and $[e]_s$ is consistent.

Al Planning

(Albert-Ludwigs-Universität Freiburg)

Cc (Ac (Ac : (Ac (Ac

April 13, 2005 37 / 60

April 13, 2005 35 / 60

33/60

 $\begin{array}{l} (\neg b_0 \vartriangleright b_0) \land \\ ((\neg b_1 \land b_0) \vartriangleright (b_1 \land \neg b_0)) \land \\ ((\neg b_2 \land b_1 \land b_0) \vartriangleright (b_2 \land \neg b_1 \land \neg b_0)) \land \\ ((\neg b_3 \land b_2 \land b_1 \land b_0) \vartriangleright (b_3 \land \neg b_2 \land \neg b_1 \land \neg b_0)) \end{array}$

Al Planning

(Albert-Ludwigs-Universität Freiburg)

(Albert-Ludwigs-Universität Freiburg)

Meaning of conditional effects >>

Increment 4-bit numbers $b_3b_2b_1b_0$.

Effects

Example

-



(and (or (on A B) (on A C)) (or (on B A) (on B C)) (or (on C A) (on A B)))

AI Planning

Succinct transition systems Schematic operators Succinct transition systems Schematic operators Example: blocks world in PDDL PDDL: operator definition ► (:action OPERATORNAME list of parameters: (?x - type1 ?y - type2 ?z - type3) (define (domain BLOCKS) precondition: a formula (:requirements :adl :typing) (:types block - object blueblock smallblock - block) <schematic-state-var> (:predicates (on ?x - smallblock ?y - block) (and <formula> ... <formula>) (ontable ?x - block) (or <formula> ... <formula>) (clear ?x - block) (not <formula>) (forall (?x1 - type1 ... ?xn - typen) <formula>)
(exists (?x1 - type1 ... ?xn - typen) <formula>)) April 13, 2005 49 / 60 (Albert-Ludwigs-Universität Freiburg) April 13, 2005 50 / 60 (Albert-Ludwigs-Universität Freiburg) Al Planning AI Planning Succinct transition systems Schematic operators Succinct transition systems Schematic operators (:action fromtable effect: :parameters (?x - smallblock ?y - block) :precondition (and (not (= ?x ?y)) <schematic-state-var> (clear ?x) (not <schematic-state-var>) (ontable ?x) (and <effect> ... <effect>) (clear ?y)) (when <formula> <effect>) :effect (forall (?x1 - type1 ... ?xn - typen) <effect>) (and (not (ontable ?x)) (not (clear ?y)) (on ?x ?y))) April 13, 2005 51 / 60 (Albert-Ludwigs-Universität Freiburg) Al Planning (Albert-Ludwigs-Universität Freiburg) Al Planning April 13, 2005 52 / 60 Succinct transition systems Schematic operators Succinct transition systems Schematic operators PDDL: problem files (define (problem blocks-10-0) (:domain BLOCKS) A problem file consists of (:objects a b c - smallblock) d e - block (define (problem PROBLEMNAME) f - blueblock) declaration of which domain is needed for this problem (:init (clear a) (clear b) (clear c) (clear d) (clear definitions of objects belonging to each type (ontable a) (ontable b) (ontable c) definition of the initial state (list of state variables initially true) (ontable d) (ontable e) (ontable f)) definition of goal states (a formula like operator precondition) (:goal (and (on a d) (on b e) (on c f)))) (Albert-Ludwigs-Universität Freiburg) AI Planning April 13, 2005 53 / 60 (Albert-Ludwigs-Universität Freiburg) AI Planning April 13, 2005 54 / 60 Succinct transition systems Schematic operators Succinct transition systems Schematic operators Example run on the FF planner Example: blocks world in PDDL edu/PS04> ./ff -o hamiltonian.pddl -f ham1.pddl ff: parsing domain file, domain 'HAMILTONIAN-CYCLE' de: (define (domain BLOCKS) ff: parsing problem file, problem 'HAM-1' defined (:requirements :adl :typing) ff: found legal plan as follows (:types block) 0: GO A B step (:predicates (on ?x - block ?y - block) 1: GO B D (ontable ?x - block) 2: GO D F (clear ?x - block) 3: GO F C) 4: GO C E 5: GO E A 0.01 seconds total time

Succinct transition systems Schematic operators

(on b a) (on a g) (on g i)))

Al Planning

```
(:action fromtable
   :parameters (?x - block ?y - block)
                                                                        (:action totable
   :precondition (and (not (= ?x ?y))
                                                                           :parameters (?x - block ?y - block)
                        (clear ?x)
                                                                           :precondition (and (clear ?x) (on ?x ?y))
                        (ontable ?x)
                                                                           :effect
                        (clear ?y))
                                                                             (and (not (on ?x ?y))
   :effect
                                                                                  (clear ?y)
     (and (not (ontable ?x))
                                                                                   (ontable ?x)))
         (not (clear ?y))
          (on ?x ?y)))
                                              April 13, 2005 57 / 60
(Albert-Ludwigs-Universität Freiburg)
                         AI Planning
                                                                       (Albert-Ludwigs-Universität Freiburg)
                                                                                                  AI Planning
                                                                                                                       April 13, 2005 58 / 60
               Succinct transition systems Schematic operators
                                                                                       Succinct transition systems Schematic operators
(:action move
                                                                        (define (problem blocks-10-0)
  :parameters (?x - block
                                                                           (:domain BLOCKS)
           ?y - block
?z - block)
                                                                          (:objects d a h g b j e i f c - block)
                                                                          (:init (clear c) (clear f)
 :precondition (and (clear ?x) (clear ?z)
                                                                              (ontable i) (ontable f)
                    (on ?x ?y) (not (= ?x ?z)))
                                                                               (on c e) (on e j) (on j b) (on b g)
 :effect
                                                                              (on g h) (on h a) (on a d) (on d i))
   (and (not (clear ?z))
                                                                          (:goal (and (on d c) (on c f) (on f j)
        (clear ?y)
         (not (on ?x ?y))
                                                                                       (on je) (on e h) (on h b)
```

)

(Albert-Ludwigs-Universität Freiburg)

```
(on ?x ?z)))
```

)

```
Al Planning
(Albert-Ludwigs-Universität Freiburg)
```

April 13, 2005 59 / 60

April 13, 2005 60 / 60