Exercise 9.1 (Probabilistic Planning – 7 credits)

Consider a system with two states $s_1$ and $s_2$ with transition probabilities expressed by the following matrix.

$$M = \begin{pmatrix}
0.9 & 0.1 \\
0.8 & 0.2 \\
\end{pmatrix}$$

So, probability of transition from $s_1$ to itself is 0.9 and to $s_2$ it is 0.1, and probability of transition from $s_2$ to $s_1$ is 0.8 and to $s_2$ itself is 0.2.

The initial state probabilities for $s_1$ and $s_2$ are expressed by the row vector

$$S = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}.$$ 

Compute the vectors $SM$, $(SM)M$, and $((SM)M)M$ representing the probabilities of the states after 1, 2 and 3 transitions.

The probabilities $P(s_1)$ and $P(s_2)$ of the system being respectively in states $s_1$ and $s_2$ asymptotically are

$$P(s_1) = 0.9P(s_1) + 0.8P(s_2)$$
$$P(s_2) = 0.1P(s_1) + 0.2P(s_2)$$

(probabilities of the possible predecessor states multiplied by the probabilities of the transitions to the state in question.) Solve the above equations, i.e. determine what the asymptotic probabilities are. (Notice that in this case the asymptotic probabilities are independent of the initial probabilities but this is not the case for all transition systems.) Can you see the connection between these probabilities and the sequence $SM$, $SM^2$, $SM^3$, $SM^4$, ...?

Exercise 9.2 (Representation – 3 credits)

Consider the following actions on state variables $A, B,$ and $C$:

$$\langle B \land C, (0.5A \lor 0.5\neg A) \land (0.5C \lor 0.5\neg C) \rangle.$$ 

Represent this operator both as a Boolean matrix and as a propositional formulae.

You may work on these assignments and submit your results in groups of two students. Make sure to clearly indicate both names on your work. You may write your answers in English or German. Please return your homework on Monday before 14:15.

Exercise marks count towards your final grade for this course, which is calculated from exercise marks (15%) and exam marks (85%).