Regression: BW2 with conditional effects

\[ 1 = \langle \top, (AonB \land A\text{clear}) \triangleright (AonT \land B\text{clear} \land \neg AonB) \rangle \]

\[ 2 = \langle \top, ((BonA \land B\text{clear}) \triangleright (BonT \land A\text{clear} \land \neg BonA) \rangle \]

\[ \text{goal} \quad AonT \land BonT \]

\[ \text{regr}1 \quad (AonT \lor (AonB \land A\text{clear})) \land BonT \]

\[ \text{regr}2 \quad (AonT \lor (AonB \lor (BonA \land B\text{clear}))) \land (BonT \lor (BonA \land B\text{clear})) \]

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Regression: properties

**Lemma D:** Let \( \phi \) be a formula, \( o \) and operator with effect \( e \), \( s \) any state and \( s' = app_o(s) \). Then \( s \models \text{regr}_r(\phi) \) if and only if \( s' \models \phi \).

**Proof:** The proof is by structural induction over subformulae \( \phi' \) of \( \phi \). We show that the formula \( \phi_r \) obtained from \( \phi \) by replacing propositions \( p \in P \) by \( (p \land \neg EPC_p(e)) \lor EPC_p(e) \) has the same truth-value in \( s \) as \( \phi \) has in \( s' \).

Induction hypothesis: \( s \models \phi'_r \) if and only if \( s' \models \phi' \).

Base cases 1 & 2, \( \phi' = \top \) or \( \phi' = \bot \): Trivial because \( \phi'_r = \phi \).

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Regression: complexity issues

1. \( \text{regr}_{(a \rightarrow p)}(p) = a \land \bot \equiv \bot \): the new set of states is empty.
   Testing that a formula \( \text{regr}_r(\phi) \) does not represent the empty set (= search is in a blind alley) is NP-hard.

2. \( \text{regr}_{(b \times a)}(a) = a \land b \): the new set of states is properly smaller.
   Testing that a regression step does not make the set of states smaller (= more difficult to reach) is NP-hard.

These tests would be useful in pruning the search tree.
Regression: complexity issues II

For a formula $\phi$, $\text{regr}_1(\text{regr}_2(\cdots \text{regr}_{n-1}(\text{regr}_n(\phi))))$ may have size $O(|\phi||o_1||o_2|\cdots|o_{n-1}||o_n|)$, i.e. the product of the sizes of $\phi$ and the operators.

Hence, the size can be in the worst case exponential in $n$ ($O(2^n)$).

Plan search: search states for regression

For regression, the search state is represented as a sequence of operators and associated formulae.

$\phi_n, o_n, \ldots, \phi_1, o_1, G$

The neighbors of the state are those obtained by regression with respect to one of the operators or by dropping out some of the last actions and associated formulae:

1. $\text{regr}_o(\phi_n), o, \phi_n, o_n, \ldots, \phi_1, o_1, G$ for $o \in O$
2. $\phi_i, o_i, \ldots, \phi_1, o_1, G$ for $i < n$ (for local search only)

Regression: complexity issues II

Split $\text{regr}_o(\phi)$ to $\phi_1, \ldots, \phi_n$ such that $\text{regr}_o(\phi) \equiv \phi_1 \lor \cdots \lor \phi_n$.

1. Transform $\text{regr}_o(\phi)$ to $\phi_1 \lor \cdots \lor \phi_n$ by suitable equivalences.

Regression planners so far have transformed $\text{regr}_o(\phi)$ to disjunctive normal form $\phi_1 \lor \cdots \lor \phi_n$ where every $\phi_i$ is a conjunction of literals.

2. Choose state variable $p$, and define

$\phi_1 = p \land \text{regr}_o(\phi)$ $\quad \phi_2 = \neg p \land \text{regr}_o(\phi)$

Plan search: distance estimates for regression

With progression we had for $I, o_1, s_1, o_2, s_2, \ldots, o_n, s_n$ the estimate

$\delta_{s_n}(G)$

With regression, we have for $\phi_n, o_n, \ldots, \phi_1, o_1, G$ the estimate

$\delta_I(\phi_n)$

Advantage: sets $D_i$ for distance estimates are computed only once, because starting state stays the same.