Implementing Prioritized merging with ASP

Julien Hué and **Eric Würbel**
LSIS-CNRS 6168,
Université du Sud Toulon -Var.
BP 132. 83957 La Garde Cedex France.
{hue, wurbel}@univ-tln.fr

O. Papini
LSIS-CNRS UMR 6168,
Université de la Méditérranée, ESIL,
163 av. de Luminy - 13288 Marseille Cedex 09. France
papini@esil.univmed.fr

Abstract

Keywords fusion, iterated revision, ASP reasoning under inconsistency.

Introduction

Fusion

fusion with preferences

papier KR06
different kinds of revision ref
prioritized merging def
different pre-orders : discrimin, leximin, linear ref
iterated revision and fusion

This paper addresses the extension of the RSF framework in two directions :the case where preferences are expressed between belief bases and the case where the belief bases are equipped with preference.
In both cases ...
The contribution : - synctactic
- implementation with ASP
We first propose syntactic prioritized fusion operations

We show that they satisfy

We then propose an implementation stemming from the Answer Sets Semantics

We illustrate the behaviour of these fusion operations according to an experimentation before concluding
intro

Background

Notations

Throughout the paper we consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. A literal is an atom or the negation of an atom. The usual propositional connectives are denoted by $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and Cn denotes the logical consequence. A *belief base* K is a finite set of propositional formulae over $\mathcal{L}$.

Prioritized merging

Let $E = \{K_1, \ldots, K_n\}$ be a multi-set of n consistent belief bases to be merged, E is called a *belief profile*. The n belief bases $K_1, \ldots, K_n$ are not necessarily different and the union of belief bases, taking repetitions into account, is denoted by $\sqcup$ and their conjunction (resp. disjunction) are denoted by $\bigwedge$ (resp. $\bigvee$). For the sake of simplicity, we denote by K the belief profile consisting of the singleton $\Psi = \{K\}$.
prioritized merging def
different pre-orders : discrimin, leximin, linear ref
iterated revision and fusion
properties
basic postulates
(PMon)
(Cons)
(Taut)
(Opt)
(IS)
(Add)
they show that a prioritized merging operator satisfying the basic postulates satisfy the KM, AGM reformulated postulates (R1)-(R4),
with Add it satisfies (R5) and (R6)

Answer Set Programming

A *Normal logic program* is a set of rules of the form $c \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$ where $c, a_i (1 \leq i \leq n), b_j (1 \leq j \leq m)$ are propositional atoms and the symbol *not* stands for *negation as failure*. Let r be a rule, we introduce $head(r) = c$ and $body(r) = \{a_1, \cdots, a_n, b_1, \cdots, b_m\}$. Furthermore, let $body^+(r) = \{a_1, \cdots, a_n\}$ denotes the set of positive body atoms and $body^-(r) = \{b_1, \cdots, b_m\}$ the set of negative body atoms, it follows $body(r) = body^+(r) \cup body^-(r)$. Moreover, r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative body atoms in the body of r.

A set of atoms X is *closed under* a basic program Π iff for any rule $r \in \Pi$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program Π is denoted by $CN(\Pi)$.

The *reduct* or Gelfond-Lifschitz transformation (0), Π^X of a program Π relatively to a set X of atoms is defined by $\Pi^X = \{r^+ \mid r \in \Pi \textit{ and } body^-(r) \cap X = \emptyset\}$.

A set of atoms X is an *answer set* of Π iff $CN(\Pi^X) = X$.

ASP solvers

In the last decade, answer set programming has been considered as a convenient tool to handle non-monotonic reasoning. Moreover, several efficient systems, called ASP solvers, have been developed for computing answer sets, Smodels (**?**), XSB (**?**), DLV (**?**), NoMore (0), ASSAT (0), CMODELS (0), CLASP (**?**).

In order to extend the expressivity and the efficiency of ASP solvers, logic programs have been extended with new statements (0):

- *domain definitions* allow for compactly encoding the possible values in a given domain, for example the declarations $\#domain\ possible(X)$, $possible(1..n)$. ensure that every occurrence of the variable X will take a value from 1 to n.
- *domain restrictions*: can be added in some rules. For example, the rule $short(X) \leftarrow size(Y), X < Y$, the rule is only instantiated for X and Y such that $X < Y$.
- *cardinality optimization*: make possible to express that at most, respectively at least, some atoms should appear in the answer sets. For example the rule $h \leftarrow k\ \{a_1, \ldots, a_n\}\ l$ expresses that at least k atoms and at most l atoms among $\{a_1, \ldots, a_n\}$ should appear in the answer sets.
- *optimization statements*: allow for selecting among the possible answer sets, the ones that satisfy optimization statements like $minimize\{.\}$ or $maximize\{.\}$. For example, the statement $minimize\{a_1, \cdots, a_n\}$ allows for selecting the answer sets with as few of the given atoms $\{a_1, \cdots, a_n\}$ as possible.

background

Prioritized Removed Sets Fusion

We here aim at building a merging operation which respects the preferences expressed over belief bases. In the context of Removed Sets Fusion, the preferences can be interpreted as a strategy which removes as few formulas as possible in high-ranked belief bases in order to restore consistency.

We will consider, for the rest of this paper, the belief profile $E = \{K_1, \ldots, K_n\}$ with IC representing constraints. The preferences expressed on this belief profile will be $K_1 < \cdots < K_n$. This constitutes a naming convention and all other cases can come down to this one thanks to a permutation.

A potential Removed Sets should be considered according to the number of formulas it removes in each belief bases. In order to compare potential Removed Sets, we thus need the following pre-order:

Definition 1 *The number of formulas removed by a potential Removed Sets X of E constrainted by IC to a belief base K_i is defined as $p^i_X = |X \cap K_i|$. We consider the sequence $(p^1_X, \ldots, p^n_X)$ of every p^i_X on the profile.*

Let X and X' be two potential Removed Sets of E constrainted by IC, the $\leq_{lexipref}$ pre-order is defined the following way:

$$X \leq_{lexipref} X' \textit{ iff } (p^1_X, \ldots, p^n_X) \leq_{lex} (p^1_{X'}, \ldots, p^n_{X'}).$$

We now can define Removed Sets of E constrainted by IC according to the $lexipref$ strategy as the potential Removed Sets of E constrainted by IC which are preferred in the sense of $\leq_{lexipref}$:

Definition 2 *Let X be a set of atoms of E and X' be a potential Removed Set of E constrainted by IC, X is a Removed Set of E constrainted by IC according to $lexipref$ iff the following conditions are respected: **(i)** X is a potential Removed Set of E constrainted by IC; **(ii)** $\forall X'$, $X' \not\subset X$; **(iii)** $\forall X'$, $X' \not<_{lexipref} X$.*

The set of every Removed Sets of E constrainted by IC according to $lexipref$ is denoted by $\Delta^{RSF}_{lexipref,IC}(E)$. The merging operation based on the $lexipref$ strategy is defined as the union of the consistent subsets of formulas constructed with the element of $\Delta^{RSF}_{lexipref,IC}(E)$.

Definition 3 *The merging operation of E constrainted by IC according to the $lexipref$ strategy is defined the following way:*

$$\Delta^{RSF}_{lexipref,IC}(E) = \bigvee_{X \in \mathcal{F}_{lexipref,IC}\mathcal{R}(E)} \{((K_1 \sqcup \cdots \sqcup K_n) \backslash X) \sqcup IC\}.$$

We will illustrate this definition with the help of an example.

Example 1 *We consider the belief profile $E = \{K_1, K_2, K_3\}$ s.t. $K_1 < K_2 < K_3$ and $IC = \top$ with $K_1 = \{a\}$, $K_2 = \{\neg a \vee b, \neg a \vee c\}$ and $K_3 = \{\neg b, \neg c\}$.*

Some of the potential Removed Sets, including the set-theoretically minimal ones, are presented in the next table with their associated $(p^1_X, \ldots, p^n_X)$ sequence:

potential Removed Sets X	p^1_X	p^2_X	p^3_X
$\{a\}$	*1*	*0*	*0*
$\{a, \neg c\}$	*1*	*0*	*1*
$\{a, \neg b\}$	*1*	*0*	*1*
$\{a, \neg b, \neg c\}$	*1*	*0*	*2*
$\{\neg a \vee b, \neg a \vee c\}$	*0*	*2*	*0*
$\{\neg a \vee b, \neg c\}$	*0*	*1*	*1*
$\{\neg a \vee c, \neg b\}$	*0*	*1*	*1*
$\{\neg b, \neg c\}$	*0*	*0*	*2*

It is easy to see that the potential Removed Set which is minimal according to $\leq_{lexipref}$ is $\{\neg b, \neg c\}$. It is even preferred to the $\{a\}$ which removes less formulas and would be preferred according to the $\leq_\Sigma$ pre-ordering.

As shown in (0), there is several ways to consider merging when preferences are expressed. Especially, it can be treated as a iterated revision problem.

Prioritized merging as an iterated revision operation

We will here define two iterated revision operator based on Removed Sets Revision operation. We define the first operation in the naive direction, from the less preferred bases to the most preferred ones. This operation, we call $\Delta^{PRSF}_{\alpha,IC}$, is not correct from our point of view. We thus define another operation, we call $\Delta^{PRSF}_{\beta,IC}$, which operates the successive revision operations in the opposite direction.

The $\Delta^{PRSF}_{\alpha,IC}$ iterated revision

The operation $\Delta^{PRSF}_{\alpha,IC}(E)$ deals with the priotized merging of E by successively revising the different belief bases from the less preferred to the most preferred.

Definition 4 *The $\Delta^{PRSF}_{\alpha,IC}(E)$ is defined by:*

$$\Delta^{PRSF}_{\alpha,IC}(E) = (((K_n \circ_{RSR} K_{n-1}) \circ_{RSR} \cdots \circ_{RSR} K_1) \circ_{RSR} IC).$$

However, the behaviour of the $\Delta^{PRSF}_{\alpha,IC}(E)$ operation is not satisfactory as illustrated by the following example.

Example 2 *We come back to the example 1. In that case, we have $\Delta^{PRSF}_{\alpha,IC}(E) = (K_3 \circ K_2) \circ K_1$. In extenso, $K_3 \circ K_2 = K_3 \sqcup K_2$ and $(K_3 \circ K_2) \circ K_1 = \{a, \neg a \vee c, \neg b\} \cup \{a, \neg a \vee b, \neg c\} \cup \{a, \neg a \vee b, \neg a \vee c\} \cup \{a, \neg b, \neg c\}$. We can see that the belonging of a formula to a certain base is lost during the revision operation and the preference is lost at the same time. This leads to a result where preferences are not taken into account correctly anymore.*

Thus, to correct this problem, we propose another operation which operate the successive revisions in the other direction — from the most preferred belief bases, to the less preferred ones.

The $\Delta^{PRSF}_{\beta,IC}$ iterated revision

Definition 5 *The $\Delta^{PRSF}_{\alpha,IC}(E)$ is defined by:*

$$\Delta^{PRSF}_{\beta,IC}(E) = (K_n \circ_{RSR} (K_{n-1} \circ_{RSR} \cdots \circ_{RSR} (K_1 \circ_{RSR} IC))).$$

The behaviour of the $\Delta^{PRSF}_{\beta,IC}$ is closer to our expectations as shown by the 1 example.

Example 3 *Considering the example 1 again. The $\Delta^{PRSF}_{\beta,IC}$ behaves as follows: $K_2 \circ K_1 = K_2 \sqcup K_1$ and $K_3 \sqcup (K_2 \circ K_1) = \{a, \neg a \vee b, \neg a \vee c\}$.*
It obtains the same result as $\Delta^{RSF}_{lexipref,IC}(E)$.

Actually, the $\Delta^{PRSF}_{\beta,IC}$ operator and the $\Delta^{RSF}_{lexipref,IC}(E)$ operator leads to the exact same result in every cases.

Proposition 1

$$\Delta^{PRSF}_{\beta,IC}(E) = \Delta^{RSF}_{lexipref,IC}(E).$$

Moreover, the $\Delta^{PRSF}_{\beta,IC}(E)$ is more interesting from the complexity point of view than the $\Delta^{PRSF}_{\alpha,IC}(E)$. Let n be the number of bases and m the maximum number of formulas into a base. $\Delta^{PRSF}_{\beta,IC}(E)$ is $\mathcal{O}(n \times 2^m)$ while $\Delta^{PRSF}_{\alpha,IC}(E)$ is $\mathcal{O}(2^{m \times n})$.

Implementation of the $lexipref$ strategy

We here propose an implementation of our $\Delta^{RSF}_{lexipref,IC}(E)$ operation based on the translation of the merging problem into a logic program with stable model semantic. Our translation is based on the work presented in (0).

This translation is in two parts: the first one computing the potential Removed Sets, the second one selecting among them the potential Removed Sets according to the $lexipref$ strategy.

The first part is already presented in (0). The generation of every Potential Removed Sets is based on the generation of every interpretation. It introduces new atoms called rule atoms. For a formula f, the rule atom r_f is deduced if the formula is not satisfied by the interpretation. The program generating all the interpretations and the corresponding sets of rule atoms is denoted $\Pi_{E,IC}$.

It has been shown in the article cited *supra* that there is a one-to-one correspondance between stable models of $\Pi_{E,IC}$ and the potential Removed Sets of E constrainted by IC. Based on this result, we can translate the notion of preference between potential Removed Sets into a preference between stable models.

Definition 6 *Let X and X' be two stable models of $\Pi_{E,IC}$. The $\leq_\Sigma$ pre-order is defined the following way:*

$$X \leq_{lexipref} X' \textit{ iff } (p^1_{(X \cap R^+)}, \ldots, p^n_{(X \cap R^+)}) \leq_{lex} (p^1_{(X' \cap R^+)}, \ldots, p^n_{(X' \cap$$

The potential Removed Sets are compared based on the number of formulas removed in each belief base. The stable models can be compared based on the number of rule atoms representing those formulas. This is the usefulness of rule atoms. It leads to the definition of preferred stable models of $\Pi_{E,IC}$ according to the $lexipref$ strategy.

Definition 7 *Let X be a set of atoms of E and X' be a stable model of $\Pi_{E,IC}$, X is a preferred stable model of $\Pi_{E,IC}$ according to the $lexipref$ strategy iff the following conditions are respected: **(i)** X is a stable model of $\Pi_{E,IC}$; **(ii)** $\forall X'$, $X' \not\subset X$; **(iii)** $\forall X'$, $X' \not<_{lexipref} X$.*

The problem consisting in determining among the stable models those which are the preferred ones is solved through a set of logic programming statement. The predicate $size(I, J)$ [1] represents the fact that J formulas are coming from K_I in the potential Removed Set. $size(I, J)$ is computed by the following rule which is introduced for every base K_I and every possible U from 1 to m which is the maximum cardinality of a belief base in the profile E.

$$\Pi_E^{lexipref,size} = \{\gamma_1 \ : \ size(V,U) \leftarrow U \ \{f_1^V, \ldots, f_m^V\} \ U.\}$$

Therefore the complete program computing the result of $\Delta^{RSF}_{lexipref,IC}(E)$ is the following: $\Pi^{lexipref}_{E,IC} = \Pi_{E,IC} \cup \Pi_E^{lexipref,size} \cup minimize[\ size(1,1) = 1 \times (m+1)^{n-1}, size(1,2) = 1 \times (m+1)^{n-1}, \ldots, size(i,1) = 1 \times (m+1)^{n-i}, size(i,2) = 2 \times (m+1)^{n-i}, \ldots, size(n,m) = m\ ]$.

The stable models of $\Pi^{lexipref}_{E,IC}$ are the preferred stable models $\Pi_{E,IC}$ according to the $lexipref$ strategy. Moreover, it computes exactly the expected Removed Sets. Formally:

Proposition 2 *The set of Removed Sets of $\Pi^{lexipref}_{E,IC}$ is the set of preferred stable models of $\Pi_{E,IC}$ according to the $lexipref$ strategy.*

Example 4 *We will here present the implementation for our example. The profile E is consisting in three belief base $K_1 = \{a\}$, $K_2 = \{\neg a \vee b, \neg a \vee c\}$ and $K_3 = \{\neg b, \neg c\}$.*

$$\Pi_{E,IC} = \left\{ \begin{array}{ccc} a \leftarrow not\ a'. & a' \leftarrow not\ a. & b \leftarrow not\ b'. \\ c \leftarrow not\ c'. & c' \leftarrow not\ c. & b' \leftarrow not\ b. \\ r_a^1 \leftarrow a'. & r^2_{\neg a \vee b} \leftarrow a, b'. & r^2_{\neg a \vee c} \leftarrow a, c'. \\ r^3_{\neg b} \leftarrow b. & r^3_{\neg c} \leftarrow c. & \end{array} \right\}$$

$$\Pi_E^{lexipref,size} = \left\{ \begin{array}{cc} size(1,1) \leftarrow 1\{r_a^1\}1. & size(2,1) \leftarrow 1\{r^2_{\neg a \vee b}, r^2_{\neg a \vee c}\}1. \\ size(2,1) \leftarrow 2\{r^2_{\neg a \vee b}, r^2_{\neg a \vee c}\}2. & size(3,1) \leftarrow 1\{r^3_{\neg b}, r^3_{\neg c}\}1. \\ size(3,2) \leftarrow 2\{r^3_{\neg b}, r^3_{\neg c}\}2. & \end{array} \right\}$$

$minimize[size(1,1) = 9, size(2,2) = 6, size(2,1) = 3, size(3,2) = 2, size(3,1) = 1]$.

which has the following stable models (with, beetween brackets, their associated weight):

$$\{a', b', c', r_a^1, size(1,1)\} \quad (9)$$
$$\{a', b', c, r_a^1, r^3_{\neg c}, size(1,1), size(3,1)\} \quad (10)$$
$$\{a', b, c', r_a^1, r^3_{\neg b}, size(1,1), size(3,1)\} \quad (10)$$
$$\{a', b, c, r_a^1, r^3_{\neg b}, r^3_{\neg c}, size(1,1), size(3,2)\} \quad (11)$$
$$\{a, b', c', r^2_{\neg a \vee b}, r^2_{\neg a \vee c}, size(2,2)\} \quad (15)$$
$$\{a, b', c, r^2_{\neg a \vee b}, r^3_{\neg c}, size(2,1), size(3,1)\} \quad (4)$$
$$\{a, b, c', r^2_{\neg a \vee c}, r^3_{\neg b}, size(2,1), size(3,1)\} \quad (4)$$
$$\{a, b, c, r^3_{\neg b}, r^3_{\neg c}, size(3,2)\} \quad (2)$$

The stable model with the minimal weight is $\{a, b, c, r^3_{\neg b}, r^3_{\neg c}, size(3,2)\}$ which correspond to the Removed Sets $\{\neg b, \neg c\}$ of E according to the $lexipref$ strategy.

Experimentation

We conducted an experimentation $\cdots$
experimentation

conclusion

We provided a frmamework $\cdots$
conclusion

[1] Variable are represented by words starting by an uppercase letter