Outline

1. Introduction
2. Theoretical Results
3. Experimental Results
4. Conclusion
Optimal sequential planning

$= A^*$ (or similar)

+ admissible heuristic

(mostly)
Folklore

Everybody knows:

If a heuristic has constant absolute error, $A^*$ requires a linear number of node expansions.
Actually, state-of-the-art optimal sequential planners are not much better than breadth-first search.

Experiments of Helmert, Haslum & Hoffmann (2007)

- BFHSP solved 37 tasks
- $A^* + h^{\text{max}}$ solved 46 tasks
- $A^* + h^{\text{PDB}}$ solved 54 tasks
- blind search solved 42 tasks
Are our heuristics bad?

Two possible explanations:

- Our heuristics aren’t that good.
- There is something fishy going on.

(Or both.)
Everybody knows:

If a heuristic has constant absolute error, A* requires a linear number of node expansions.

But...

This relies on several assumptions:

- fixed branching factor
- only a single goal state
- no transpositions

These assumptions do not hold in any common planning task!
Almost perfect heuristics differ from the perfect heuristic $h^*$ only by an additive constant:

**Definition**

Define heuristic $h^* - c$ (for $c \in \mathbb{N}_1$) as

$$(h^* - c)(s) := \max(h^*(s) - c, 0)$$

→ unlikely to be obtainable in practice
How many nodes must $A^*$ expand for a planning task $\mathcal{T}$, given an almost perfect heuristic $h^* - c$?

**Definition**

$$N^c(\mathcal{T}) := \text{number of states } s \text{ with } g(s) + (h^* - c)(s) < h^*(\mathcal{T})$$

$\Rightarrow$ If this number grows fast with scaling task size, we have a problem.

**Objective**

Results for $N^c(\mathcal{T})$ for IPC domains

→ Focus on domains in APX
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Our goal

Find sequence \((\mathcal{T}_n)\) of scaling tasks for which \(N^c(\mathcal{T}_n)\) grows exponentially, even for small values of \(c\).
\( \mathcal{T}_n \): Task with \( n \) balls

\( S_n \): Total number of reachable states of \( \mathcal{T}_n \)

\[ S_n = 2 \cdot (2^n + 2n2^{n-1} + n(n-1)2^{n-2}) \]
Let $n \in \mathbb{N}_0$ with $n \geq 3$.

If $n$ is even, then

- $N^1(T_n) = N^2(T_n) = \frac{1}{2}S_n - 3$
- $N^c(T_n) = S_n - 2n - 2$ for all $c \geq 3$.

If $n$ is odd, then

- $N^1(T_n) = N^2(T_n) = S_n - 3$
- $N^c(T_n) = S_n - 2$ for all $c \geq 3$. 
Gripper

Proof

Proof sketch

- \( n \) is even
  - states with an even number of balls in each room
    - basically all are part of an optimal plan
  - states with an odd number of balls in each room
    - all are part of plans of length \( h^*(\mathcal{T}_n) + 2 \)

- \( n \) is odd
  - basically all states are part of an optimal plan
**MICONIC-SIMPLE-ADL**

- $\mathcal{T}_n$: Task with $n$ passengers (and $n + 1$ floors)
- $S_n$: Total number of reachable states of $\mathcal{T}_n$

$$S_n = 3^n(n + 1)$$
For all $c \geq 4$:

$$N^c(T_n) = S_n - (2^n - 1)(n + 1).$$
Blocksworld

$\mathcal{I}_n$: Task with $n$ blocks ($n \geq 2$)
Blocksworld

Theorem

\[ N^1(T_n) = 4 \cdot \sum_{k=0}^{n-3} B_k + 3B_{n-2} + 1 \]

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### Question

#### Theoretical results

There exist task families for which the number of states expanded by $h^* - c$ grows exponentially, even for small $c$.

#### Interesting question

Can we observe this behaviour in practice?

→ Experiments with IPC tasks
Values $N^c(T)$ are defined in terms of $h^*$. Usually $h^*$ cannot be determined efficiently.

Naive way of computing $N^c(T)$

- Completely explore the state space of $T$.
- Search backwards from the goals to determine the $h^*(s)$ values.

→ Observation: Generating all states is not necessary.
Search space

- **goal node**
- **node belongs to** $N^c(T)$
- **node does not belong to** $N^c(T)$

$h^*(T)$

$h^*(T) + c - 1$

↝ Poster session: today, 6:00-9:30 PM
## Results

### Blocksworld

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## Results

### Gripper

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## Results

### Logistics (IPC 2)

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## Results

### Miconic-Strips

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Dismal prospects

Depressing theoretical and experimental results

- Other (similar) search techniques cannot perform better than $A^*$.
- With other (real) heuristics it gets worse.
What is the cause of this behaviour?

Main problem

- many independently solvable subproblems which can be arbitrarily permuted
- many possible orders

Why is this not common knowledge?
→ does not happen in 15-Puzzle, Rubik’s Cube, etc.
Some possible conclusions:

**Conclusion?**

We need heuristics that are better than almost perfect.  
How feasible is this?

**Conclusion?**

We need more search enhancements.  
Look to domain-dependent search for guidance?
What can the search community offer us?

Domain-specific search enhancements for Sokoban (Junghanns and Schaeffer, 2001):

- transposition table ➞ irrelevant to analysis
- move ordering ➞ irrelevant to analysis
- deadlock tables ➞ irrelevant to analysis
- tunnel macros ➞ generalizable?
- goal macros ➞ incomplete
- goal cuts ➞ incomplete
- pattern search ➞ heuristic improvement
- relevance cuts ➞ incomplete
- overestimation ➞ suboptimal
- rapid random restart ➞ irrelevant to analysis

⇝ Poster session: today, 6:00-9:30 PM
General search enhancements

Some techniques that might work in general:

- partial-order reduction
- symmetry elimination
- problem simplification
What do the results mean for us?

Some alternative conclusions:

**Conclusion?**

Heuristic search doesn’t cut it.
What about more global reasoning methods, such as SAT planning, or symbolic exploration techniques like breadth-first search with BDDs?

**Conclusion?**

Optimal planning, beyond a certain point, is too hard.
We can hope to scale a bit better than blind search, but not very far. Maybe study near-optimal planning in a more principled way instead?
Thank you for your attention!